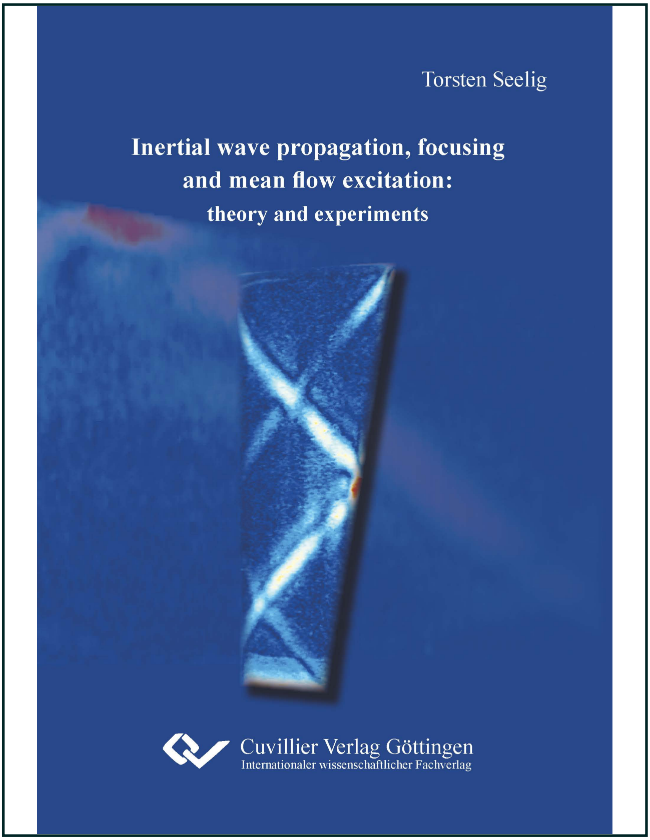
Torsten Seelig (Autor)

Inertial wave propagation, focusing and mean flow excitation

theory and experiments



https://cuvillier.de/de/shop/publications/6760

1 Introduction

1.1 Motivation and formulation of the problem

Mean zonal motions/flows or currents are associated with the general circulation of the atmosphere or the oceans. The general circulation on the other hand is related to the dynamics of climate. This is the variability of fields of climate elements (wind, temperature, salinity, ozone etc.) temporally averaged over periods ranging from months to thousands of years. In meteorology usually an average over a period of 30 years is used. Traditionally, observational and theoretical studies concentrate on the zonal (latitudinal) mean flow. This has its origin in the linear plane wave theory in which the flow field is split into zonal mean and meridionally (longitudinally) dependent eddy component [Hol04]. The meridionally dependent components can again be categorised into circulations which have shorter periods. Quasi-stationary circulations, which vary little in time, imply stationary Rossby waves and on a shorter length scale the North Atlantic oscillation. Subtropical circulations are seasonally reversing and well-known in the atmospheric wind circulation as Monsoons but have a counterpart in oceanography in form of the Indian Monsoon Currents. Finally there are various subseasonal and interannual circulations which are combined to low-frequency variability. The Tropical subseasonal variability or the Interannual Atlantic Meridional Overturning Circulation are examples of the latter variability.

Well known is also atmospheric variability with periods of years. Most prominent is the atmospheric QBO and the (Atlantic) Equatorial Deep Jets (EDJs). The first one is an equatorial quasi-periodic oscillation of easterly and westerly wind propagating downward in time. The pattern ranges from the lower stratosphere to the upper troposphere and has a mean period of approximately 27 months [AHL87, BGD⁺01]. The second ones are alternating equatorial zonal currents measured in all three equatorial oceans. They range from ocean's surface to approximately 2500 m depth and oscillate with mean periods of approximately 4.5 years and downward phase propagation of 132(±12) m/yr [JZ03, BPHK08, BFH⁺11]. Today in current research it is accepted that the QBO is mainly driven by waves. But what are waves, which types occur in the atmosphere and which of them are relevant? In her thesis Astrid M.M. Manders described it as follows:

> “ *Waves own their existence to a restoring force, acting on a perturbation of a background equilibrium state. This perturbation is forced back to its original position, overshoots since it has finite velocity when reaching it, and the sign of restoring force reverses. This process is repeated, resulting in an oscillation. The type of wave depends on the particular restoring force. For geophysical fluids, this can be gravity, for example for surface water waves, but also surface tension (small capillary waves at the surface), the pressure gradient (sound waves), or Coriolis force (Poincaré waves, Rossby waves). The wave energy propagates, but the particles of the fluid just oscillate and pass the energy through, after which they return to their equilibrium position (in linear approximation). Which waves are relevant depends on the scale and specific setting one is interested in. On the scale of the ocean, Rossby waves are visible, whereas for a ship the wind-generated surface waves are most important.”* [Man03]

Vertically upward propagating gravity waves are relevant for the QBO [PM78, LH68, HL72, WS06]. They dissipate and lead to vertical momentum-flux divergence that drives the mean zonal motion. Somewhat surprising, the effect of rotation is neglected even though it is known that vertical momentum transport in the tropical atmosphere and oceans is *not* only caused by vertically propagating gravity waves. Eastward propagating Kelvin waves (provide westerly momentum) and westward propagating Rossby waves (or mixed Rossby-gravity waves provide easterly momentum) are believed to play an equally important role [Dun97]. It is obvious that rotation plays a main role in the generation of both wave types. Equatorial Kelvin waves are equatorially trapped due to their earths β-effect. Rossby waves can be trapped due to shear in rotating fluids and are in fact a subset of inertial waves [Ros39, LPZK12]. Inertial waves or gyroscopic waves [BBSB34, LM78, Gre90] come into existence due to small perturbations in a homogeneous rotating fluid that is stratified by constant angular momentum. The wave balances Coriolis force against the pressure gradient force in the rotating system. The mechanisms which drive the EDJs are less well understood compared to the QBO.

Therefore the thesis concentrates on three main topics to cast some light on (i) inertial waves propagating in the interior of a rotating homogeneous fluid, (ii) the effect of inertial wave reflection especially on a sloped wall and (iii) the interaction of inertial waves with the zonal flow in a sense of the QBO which is driven due to dissipation of gravity waves.

1.2 This thesis

The thesis consists of nine chapters. Fundamental theoretical aspects of inertial waves are addressed in chapter 2. The dispersion relation and other important properties as phase and group velocity as well as the reflection of inertial waves from rigid boundaries and the effect of focusing of wave energy are derived; followed by a short introduction of inertial rays, attractors, angular momentum mixing and the Taylor-Proudman theorem.

Chapter 3 outlines the current state of research. Starting point is the remarkable experiment of Plumb and McEwan [PM78] that explains the generation of the QBO. It is a non-rotating and density-stratified experiment that generates only gravity waves which drive the slow zonal/azimuthal oscillation due to vertical momentum-flux divergence. A lack of this experiment is the absence of rotation. In view of thus we motivate a rotating angular momentum-stratified experiment that yields also to the history of rotating and librating[1] flows.

Chapter 4 describes in detail the experimental setup. First the design is motivated and its new and unique mechanical design is presented. Detailed technical aspects of the apparatus that has been built new in the years 2011/2012 at the BTU CS and its sensors and control unit are further specified.

Chapter 5 introduces the measuring techniques that have been used. Qualitative as well as several quantitative methods can be utilised to measure the flow. The qualitative kalliroscope technique to visualise the flow, their mechanism and technical setup is presented in detail. Two-dimensional velocity fields have been measured by the technique of PIV. Both technique and setup are also described.

Without an appropriate post-processing inertial waves would probably remain undiscovered in the present experiment. All important information about the spatial structure of inertial waves, their propagation, reflection and some other mechanisms such as the efficiency of wave excitation have been filtered out by applying the harmonic analysis. Their mathematical derivation and assimilation to the laboratory experiment and further the modus operandi is addressed in chapter 6.

In chapter 7 inertial waves studied in the laboratory are presented. Typical flow pattern observed in a radial-axial cross-section from the laboratory frame are shown. Inertial waves are forced in a homogeneous liquid due to the libration of the outer cylinder together with lids. In this case, the excitation of waves is very efficient and the flow pattern can be seen by naked eye. Further, weak viscous experimental and inviscid theoretical results are compared both for (i) frustum and (ii) outer cylinder together with lids libration. Wave pattern, wave reflection and the formation of wave attractors produced by the effect of focusing of wave energy that counteracts viscous energy dissipation are shown. Especially an 1/1 attractor is

---

[1] A sinusoidal oscillation is superposed on the mean rotation rate, in order to excite inertial waves.

investigated in detail with both measuring techniques mentioned above. The study of mean flows in horizontal cross-sections at different levels measured in the co-rotating frame via PIV provides an overview of several types of mean zonal motions. A persistent jet of Stewartson-type exhibiting prograde mean flow has been found. Additionally a jet originating from the focusing reflection point of an 1/1 attractor exhibiting retrograde mean flow is studied. Finally the efficiency of wave excitation and the azimuthal mean flow (Stewartson-type jet) dependence on the libration amplitude is discussed.

Chapter 8 is devoted to the interaction of inertial waves with the zonal flow similar to the QBO which is driven by gravity wave dissipation. A simple model is discussed for the inertial wave driven mean flow obtained from the primitive equations. [Plu75] described the generation of a “mean zonal motion” due to momentum transport of vertically propagating gravity waves. Based on the mathematical analogy it is shown that in the meridional plane propagating inertial waves can transfer their momentum in the same manner to a sheared mean flow. By this mechanism a mean flow can develop. Even an alternating mean flow can be driven by inertial waves. The comparison of numerical results originating from numerical computations of two simple analytical model equations and long-time measurements of velocity fields concludes the study.

Finally, summarising comments and a motivation for a future project is given in chapter 9.

2 Fundamentals

2.1 Content

The dynamic of the atmosphere as well as the oceans is governed by a wide range of processes on different scales. Many of these processes are controlled by waves [Lig78, Ped03, HEWS09]. Waves transport momentum and give it locally to the environment that give rise to slowly alternating mean flows. Such mean flows play an important role both for weather and climate processes as well as the dynamics of the oceans. The mathematical and physical formulation of atmospheric and oceanic wave-driven currents are discussed in detail in various books [Gre90, Ped87, Ped03, Hol04].

Prominent wave-driven currents are mostly located in a strip around the equator ranging from latitudes of approximately 25° S to 25° N. In general there is a coupling of stratification of the fluid and the rotation of the Earth and waves excited in such an environment are called inertia-gravity waves. When the mechanisms are considered separately one find an analogy or isomorphy. The analogy between rotating and stratified flow has been discussed by many authors (for a survey see e.g [Lin55, Ver70, Vla85, RN99]).

In the present study the stratification of the fluid is neglected and the rotation and associated inertial motions instead should play the dominant role. A brief summary of the main properties of inertial waves can be found below. First, the fundamental equations which describe the motion of a fluid and characteristic numbers are presented. In the next paragraph basic properties of inertial waves are explained. For this purpose, the strip around the equator (only the northern hemisphere) is assumed to be a channel, while neglecting the curvature of the earth (equatorial f-plane). The channel rotates around its axis that points northward. Assuming a Cartesian coordinate system then the rotation axis coincides with the y-axis, the x-axis points in eastward direction and the z-axis points in the vertical direction. Further the process of angular momentum mixing is illustrated and finally the Taylor-Proudman theorem is presented which describes the fluid motion above a rotating body.