



Kai-Simon Goetzmann (Autor)

**The Power of Compromise**

Reference Point Methods and Approximation in Multicriteria  
Optimization

Kai-Simon Goetzmann

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# Contents

<b>1</b>	<b>Introduction</b>	<b>7</b>
1.1	Motivation . . . . .	7
1.2	Our contribution . . . . .	11
1.3	Preliminaries . . . . .	13
1.4	The state of the art . . . . .	19
1.5	Related concepts . . . . .	24
<b>I</b>	<b>Theory</b>	<b>33</b>
<b>2</b>	<b>Equivalence of approximability</b>	<b>35</b>
2.1	Reference point solutions and the Pareto set . . . . .	36
2.2	From Pareto sets to reference point solutions . . . . .	39
2.3	From reference point solutions to Pareto sets . . . . .	41
<b>3</b>	<b>Algorithms</b>	<b>45</b>
3.1	Approximation by weighted sum . . . . .	45
3.2	Convex optimization with linear objectives . . . . .	46
3.3	Approximation through LP rounding . . . . .	48
3.4	From pseudopolynomial algorithms to approximation . . . . .	49
3.5	Enumeration techniques . . . . .	54
3.6	Weighted disjoint hitting set . . . . .	57
<b>4</b>	<b>Maximization problems</b>	<b>65</b>
4.1	Equivalence of approximability . . . . .	66
4.2	Algorithms . . . . .	70
<b>5</b>	<b>Arbitrary reference points</b>	<b>75</b>
5.1	Equivalence of approximability . . . . .	77



- II Practice** **83**
  
- 6 Synthetic sustainability indicators** **85**
  - 6.1 The double reference point method . . . . . 88
  - 6.2 Special types of objectives . . . . . 92
  - 6.3 Determination of the parameters . . . . . 93
  - 6.4 Critique . . . . . 96
  
- 7 Evaluating genetic algorithms** **99**
  - 7.1 Algorithms . . . . . 101
  - 7.2 Performance indicators . . . . . 105
  - 7.3 Test problems . . . . . 107
  - 7.4 A comparative study . . . . . 108
  - 7.5 Critique . . . . . 122
  
- 8 Conclusion** **125**
  
- A More proofs on approximability** **127**
  - A.1 From the  $p$ -norms to the maximum-norm . . . . . 127
  - A.2 Results from Section 2.3 for  $p$ -norms . . . . . 129
  - A.3 Results from Section 3.1 for  $p$ -norms . . . . . 131
  - A.4 Results from Section 4.1 for  $p$ -norms . . . . . 132
  
- B Computing the  $\varepsilon$ -indicator** **135**
  
- Bibliography** **138**
  
- Nomenclature** **151**