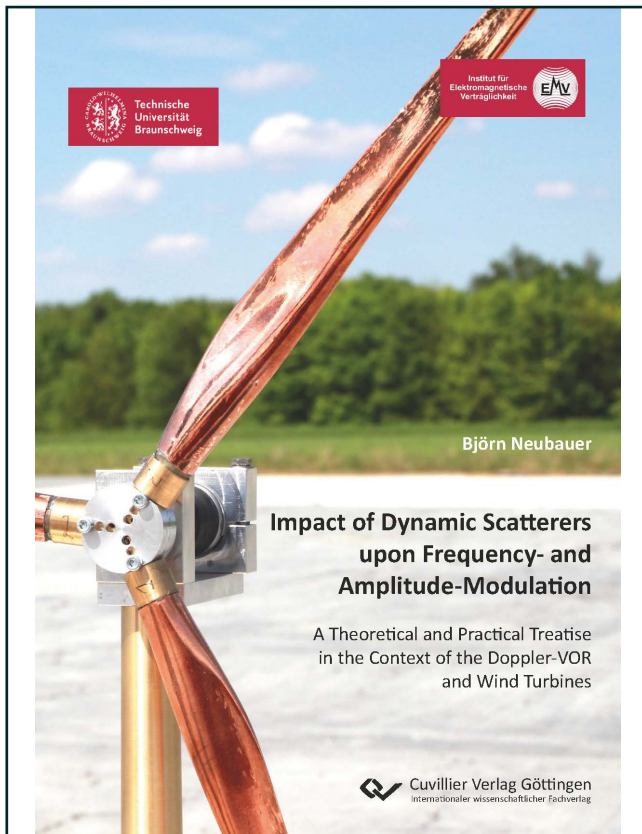




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# **Impact of Dynamic Scatterers upon Frequency- and Amplitude-Modulation**

A Theoretical and Practical Treatise in the Context of the Doppler-VOR and Wind Turbines



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