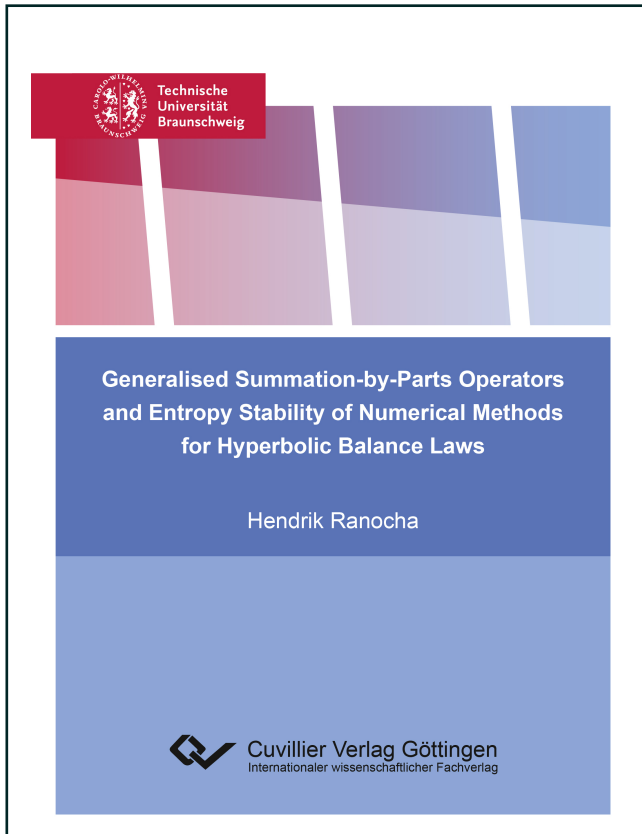




Hendrik Ranocha (Autor)

Generalised Summation-by-Parts Operators and Entropy Stability of Numerical Methods for Hyperbolic Balance Laws



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