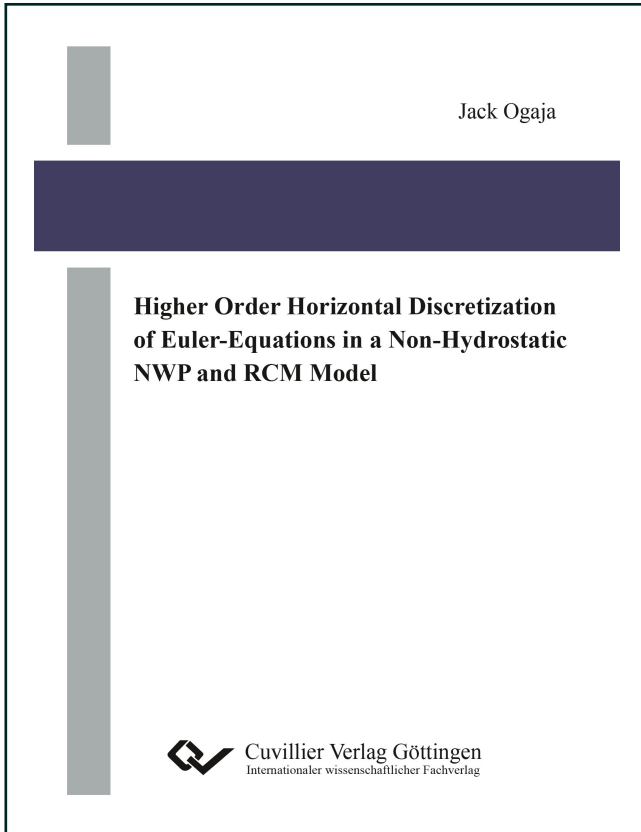




Jack Ogaja (Autor)

## **Higher Order Horizontal Discretization of Euler-Equations in a Non-Hydrostatic NWP and RCM Model**



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Telefon: +49 (0)551 54724-0, E-Mail: [info@cuvillier.de](mailto:info@cuvillier.de), Website: <https://cuvillier.de>



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