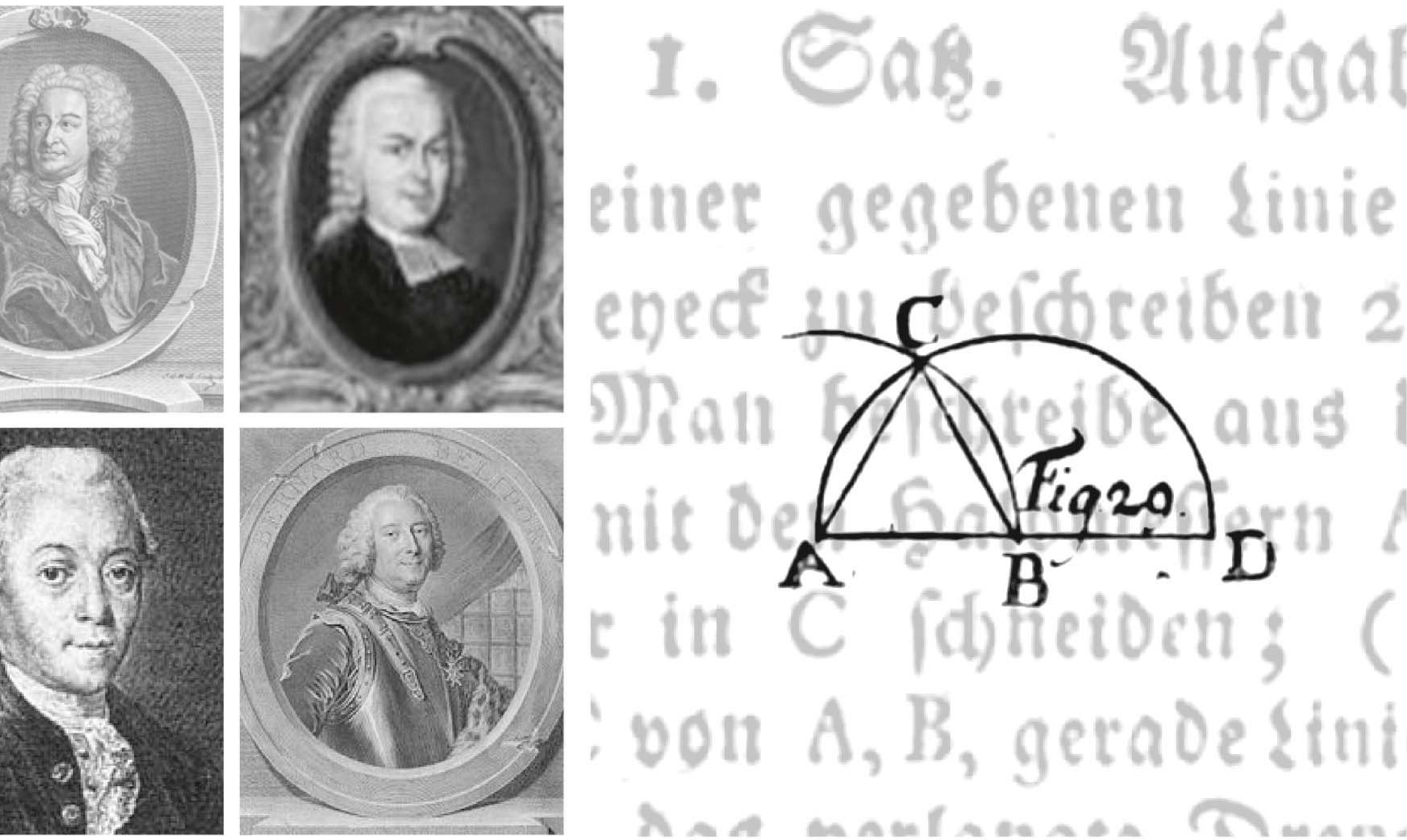
Sara Confalonieri / Desirée Kröger

Teaching the Mathematical Sciences at French and German Universities during the 18th Century



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Teaching the Mathematical Sciences at French and German Universities during the 18th Century

Sara Confalonieri and Desirée Kröger

Fakultät Mathematik und Naturwissenschaften

Arbeitsgruppe Didaktik und Geschichte der Mathematik

Bergische Universität

Gaußstraße 20, 42119 Wuppertal

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Nonnenstieg 8, 37075 Göttingen, Germany
Telephone: +49 (0)551-54724-0
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1 Introduction

During the 18th century in France and Germany a rather new kind of textbooks, which we will call the *Cours* and *Anfangsgründe* literature, was published. These textbooks were created, on the one hand, to assist the teaching of the mathematical sciences in higher education and, on the other hand, to make the reception of the teaching among students more effective. Interestingly, a structural point of view seems to be contained in the titles themselves. Indeed, "*cours*" means "course, lessons, classes", while *Anfangsgründe* means "elements, basics": the relation to teaching is explicit in French textbooks, whereas it remains implicit in the German ones. The textbooks in both languages were usually for beginners; the novelty was indeed that they were written in the respective national languages. The reformers of the Enlightenment fought for the dissemination of knowledge – what could be realized, for instance, using the national languages instead of Latin in education and books. Thereby, also those people who were not allowed to study, like women, had the possibility to learn the mathematical sciences.

The *Cours* and *Anfangsgründe* textbooks were adapted to the particular circumstances of the 18th century, namely to the way the mathematical sciences were taught at this time. Before this period, the main teaching method was rote memorization. Traditionally, knowledge was dictated without any closer examination. This changed in the context of Enlightenment, when autonomous thinking became the principal goal of education. For this purpose, the *Cours* and the *Anfangsgründe* were very useful due to a variety of respects. Indeed, they could be used as lecture notes, so that the relevant topics did not have to be dictated anymore, and, also, as memorandum, where students could afterwards search for contents.

Among the contemporary studies on the history of mathematics teaching, there is a lack of literature on the substance and development of these kinds of textbooks. In order to fill this gap, the present work[1] deals with some characteristics, with the structure, and with the contents of the *Cours* and the *Anfangsgründe* literature. Of peculiar inter-

1 This work provides some results of the project "*Traditionen der schriftlichen Mathematik und Mathematikvermittlung im deutschen und im französischen Sprachraum zwischen 1650 und 1820-Herausbildung und Differenzierung von wissenschaftlichen Disziplinen in nationalen Kontexten*", supported by the Deutsche Forschungsgemeinschaft (DFG) at the Bergische Universität, Wuppertal. The final aim of the project was not only to establish a comparison between the French and German textbooks that were used during the 18th century to teach the mathematical sciences in higher education, but also to eventually analyze the emergence of teaching traditions by retracing their possible origins in the textbooks written in Latin, especially by the Jesuits. To this purpose, together with Dagmar Mrozik, we moreover worked on a comprehensive database based on the software Archiv-Editor, developed by the DFG project "*Personendaten-Repositorium*", at the Berlin-Brandenburgische Akademie der Wissenschaften.

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est in these textbooks might be the educational transposition – that is the systematic and didactic editing of research contents (cf. Chevallard 1991, pp. 39 ff.). On the one hand, this implies some differences between research and school knowledge, which were already institutionally split during the 18th century. On the other hand, it seems that the authors of this textbooks wanted to combine these two elements, which accounts for an interaction between scientific disciplines and didactics. In addition to that, our knowledge about the educational system in the 18th century is not thorough enough. Our study of the textbooks could enlighten the situation: it could contribute to clarify the educational circumstances of how mathematical sciences were taught, and also the role of mathematics in higher education during this period. Despite the fact that there were many differences between the French and the German educational system during the 18th century, the *Cours* and the *Anfangsgründe* present several similarities, so that it is fully profitable to discuss and compare them.

First of all, we need to deal with two preliminary points to clear the ground from any possible misunderstanding. During the 18th century, as well as in the preceding centuries, the terms "*mathématique(s)*" and "*Mathematik*" were still understood in a much wider sense than we nowadays do. The French and German authors of the textbooks of this period only rarely used the terms "*sciences mathématiques*" and "*mathematische Wissenschaften*". Nevertheless, we prefer to employ, when needed, the term "mathematical sciences" rather than "mathematics" to underline its comprehensive meaning and to recall that, at that time, among the mathematical sciences were included not only the pure disciplines such as geometry, arithmetic, algebra, and analysis, but also the applied ones, such as, for instance, mechanics, optics, astronomy, civil and military architectures. In general, the pure mathematical sciences were regarded as the doctrine of the magnitudes, that is, what can be measured or calculated; whereas their instantiation in some concrete bodies were considered within the applied mathematical sciences (for more details, cf. Section 2.1.2 on page 10 and Section 3.6 on page 37).

The second preliminary point concerns the choice of the textbooks. We consider the following criteria. Firstly, the textbooks must have been written with a teaching purpose for higher education. Secondly, they must have meant to provide a complete presentation of the mathematical sciences. Whatever "complete" means depends not only on each single author, but also on the time span. Indeed, there were some shifts in Germany during the 18th century concerning the framework of the mathematical sciences. Thirdly, we only consider textbooks that are written in a national language, namely French or German. Fourthly, we focus on the 18th century.

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For the case of France, two facts must be taken into account: that the first textbooks not in Latin appeared in the first half of the 17th century and that the educational system underwent some major changes during the French Revolution. Therefore, we rather consider the textbooks written within this time lapse. With these limitations , we found around sixty textbooks. Clearly, we cannot provide a satisfactory account of all of them in this paper, so we choose a small selection including the most used ones, according to the number of their editions and to the secondary literature. In the end, this selection includes the textbooks by Bernard Forest de Bélidor (1698-1761), Nicolas-Louis de La Caille (1713-1762), Charles Étienne Louis Camus (1699-1768), Étienne Bézout (1739-1783), and Charles Bossut (1730-1814).

For the case of Germany, the *Anfangsgründe* tradition begins with Christian Wolff (1679-1754), who published his *Anfangsgründe aller mathematischen Wissenschaften* in 1710. It was often used and reprinted until 1800, a long time after his death. It was without any competition for almost fifty years, until the next generation of mathematicians published their textbooks in the second half of the 18th century: Abraham Gotthelf Kästner (1719-1800), Johann Andreas von Segner (1704-1777), Wenceslaus Johann Gustav Karsten (1732-1787), Heinrich Wilhelm Clemm (1725-1775), and Georg Simon Klügel (1739-1812). The textbooks of these authors were also the most used ones in the 18th century – as it is shown by the number of their editions and by comments in the secondary literature (cf. Kühn 1987, pp. 72 ff.). After this period, the *Anfangsgründe* seemed to be out of use, which can be explained by the changes within the German educational system. In 1810, there was the popular educational reform in Prussia. The reformers established different kinds of schools, reworked the curricula, and required new adapted textbooks. In addition to that, there was a vast increase of knowledge, mainly in analysis (differential and integral calculus) which should have been included in the new textbooks.

In the second place, it is worth to spend a few words on the peculiarities of the 18th century in Europe. For sure, one of the most distinctive features of this period is the encyclopedism, that is, the attitude of human beings towards knowledge characterized by the wish to satisfy curiosity in completest way possible, and to classify the results. Encyclopedias are the tangible embodiment of this attitude. While during the Middle Ages the clergy had the monopoly over knowledge, starting from Renaissance – and in particular from the 17th century – it became more widespread and, at the same time, more specialized. The production of scholar works increased and got fragmented into many topics. This caused, on the one hand, the creation of specific tools and, on the other hand, determined their success. These were: catalogs, bibliographies, indexes,

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analytical tables, and – above all – dictionaries and encyclopedias. Particular attention was paid to the technical knowledge that, mainly orally conveyed, was now displayed, theorized, and visually represented. Among the dictionaries and encyclopedias we count, for instance, the *Lexicon Technicum, or an Universal English Dictionary of Arts and Sciences* (1704) by Harris, the *Universal Lexicon aller Wissenschaften und Künste* (1732-1750) by Zedler, and, of course, the *Encyclopédie ou Dictionnaire Raisonné des Sciences, des Arts et des Métiers* (1751-1772) by D'Alembert and Diderot. With the latter, the encyclopedic attitude reaches without any doubt its highest point. It is composed of seventeen volumes of text and eleven volumes of figures, so that Voltaire judged it as a "*monument des progrès de l'esprit humain*". In the *Encyclopédie*, sciences, arts, and crafts – that is, the scientific and technical knowledge – play a major role. At the beginning of the publication, D'Alembert has already been a member of the *Académie des sciences* and an author of a renown mechanical treatise, while Diderot had worked in the domains of medicine and mathematics. Moreover, a good number of collaborators were the savants of the time, working in scientifical domains. D'Alembert himself wrote most of the mathematical parts (helped by La Chapelle), the mechanical parts (in which he was a specialist), and the astronomical parts (helped by Formey and Jacourt). One of the main peculiarities of this encyclopedia are the parts about the technical crafts: the authors wanted to reestablish them, since they could no longer be ignored due to technical progress. In 1777, four volumes of text and one volume of figures were added to the *Encyclopédie* by Panckoucke, who is also author of the *Encyclopédie Méthodique* (1832).

The present work is divided into three main sections. In the first two, we give a parallel overview of the *Cours* and of the *Anfangsgründe* literature, respectively. Firstly, we take a look at the circumstances in education in France and Germany in order to contextualize the textbooks. Then, we give a description of a selection of textbooks. We take care to answer the following questions: When were these textbooks published? Who were the authors? For whom were the textbooks written? What was the intention of the authors? What are the peculiar contents and structures of these textbooks? What do we know about their usage and dissemination? Unfortunately, due to the lack of sources, we cannot always give a completely satisfactory account; nevertheless, our work provides solid basis for further studies. In the third section, we give an insight into some specific topics of the *Cours* and of the *Anfangsgründe.* For this purpose we focus on four case studies: negative numbers, Pythagoras theorem, ballistics, and fortification.

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2 The *Cours* Literature

2.1 Educational Circumstances in France during the 18th Century

With regard to institutionalized science teaching, we need at first to consider that, as Brockliss points out,

> [i]n the context of seventeenth- and eighteenth-century France the term "higher education" is an anachronism. It implies the existence of a carefully articulated system of educational provision, functionally differentiated and age-specific. But at this date no such system pertained anywhere in Europe, let alone in France (cf. Brockliss 1987, p. 2).

In the following, we summarize the main characteristics of the French educational system during the 18th century.[2] We moreover specify which knowledge fields defined the term "mathematical sciences" in this period.

2.1.1 Institutions

The mathematical sciences were taught in a variety of contexts in France during the 18th century. From the most to the least attended establishments, there were: universities, colleges, military schools, technical schools, and *maisons particulières*. The mathematical sciences were also taught by private teachers. Since the latter were completely independent from the institutional context (which makes the data gathering extremely difficult), we do not report on it.

The foundation of universities in France dated back to the 12th century. On the eve of the French Revolution, there were about 25 universities with 300-400 students on average each. They were financed by private endowments (in particular by the Crown) and by the students' fees, so that they were quite prosperous overall. They had the monopoly of granting degrees in one of the three faculties, namely medicine, law, and theology. The mathematical sciences were also dealt with at universities, even though they were not directly taught. Since the colleges had absorbed the teaching of the former faculty of arts, this last faculty had been reduced to the function of delivering the necessary degrees for entering one of the three faculties. More precisely, some boards had been created to evaluate the students in the propedeutical subjects – among which mathematics was – in order to admit them.

2 The main source of the following paragraph is (Brockliss 1987).

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The colleges had generally been created more recently and in a larger number than universities. Such institutions had of course existed for a long time, but they were intended to serve as residences where underprivileged students were lodged and fed. These institutions were directly attached to universities. The colleges, as we know them during the 18th century, began to evolve during the second half of the 15th century, when some of the residences in Paris started to run classes in competition with the Faculty of Arts. Shortly before the French Revolution, of the 348 colleges only 171 offered a complete teaching that included also the last year of philosophy: they were called *collèges de plein exercice* (cf. Brockliss 1987, p. 22). During the 17th century, they were generally more often attended than universities, but afterwards, due to the overall increase of the educational offers, a period of decline started. The colleges were run both by seculars (cf. Brockliss 1987, pp. 23 and 481) and by teaching orders, like the Jesuits (until their banishment in 1762), the Oratorians, the Benedectines, and, to a lesser extent, by some others. Most of the colleges in the 18th century inherited the boarding schools feature of their predecessors. They were mainly intended for those students who wanted to continue their studies at university and could only award a degree if they were affiliated to a university. As already mentioned above, the liberal arts (in particular languages) and philosophy were their major teaching subjects. Mathematical contents were implemented only during the last philosophy year, when the main topics of elementary pure mathematics were taught to students without prior mathematical knowledge. This also included a large amount of physics, so that a considerable number of topics had to be dealt with within a short time span.

The military schools were instead far more recently created, namely, around the second half of the 18th century. They derived from the practice common in each company of the army to train a certain number of young nobles to become the future army officers. This instruction was then taken over by the government. The military schools were founded in small towns, usually where a college previously existed, and were the *élite* basis of the future *académies*. Since they were in most cases restructured colleges, they inherited some of their characteristics: they were, for instance, boarding schools run by regular teaching orders, especially by the Benedectines. In contrast with the colleges, the whole range of the mathematical sciences was taught in the military schools. Classics and philosophy were also taught, as well as equitation and military tactics – an instruction that only these institutions delivered.

In technical schools, like the École Royale des Ponts et Chaussées (1775) and the École des Mines (1783), and in the *académies* or *maisons particulières* some mathematical teaching were also imparted. As the military schools, they were founded dur-

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ing the second half of the 18th century and, complementary to them, they were meant to satisfy the demand for institutional instruction in the applied mathematical sciences.

To sum up, during the 18th century the mathematical sciences were thoroughly taught in two kinds of institutions, namely colleges and military schools. Nevertheless, only in the military schools the teaching included a wide spectrum of topics and only in some of these schools a high level of contents was reached. If we exclude elementary military schools, where only an elementary teaching in the mathematical sciences was delivered, there were mainly three kinds of military schools that instructed the future officers in higher mathematical topics: the one for the navy corps, the one for the artillery corps, and the one for the military engineer corp.

The military schools for the navy were a long-lasting institution. They were founded in 1689 by royal order and lasted almost until the French Revolution. Many professors and students were also part of the newly-founded Académie Royale de Marine in Brest (1752). Despite all the premises, the navy schools never attained an outstanding level. The students lacked discipline, their theoretical studies were often interrupted by the wars (and also by the sea service during peacetime), and in general, the navy in France at that time was not as renowned as the military corps on the mainland. In 1716, the *élite* corps of the Gardes du Pavillon was created within the navy. In 1763, after the defeat against England during the Seven Years' War, the teaching for the navy was reformed. Étienne Bézout was commissioned with the renewal. One year later, he wrote the first volume of his *Cours* and was appointed *examinateur*. Indeed, to guarantee a higher standard for the officers' education, examinations were established in order to admit the students and to let them pass to the subsequent years (as already was the case for the École du Génie). Nevertheless, this change did not have the expected positive impact. From 1771 to 1774, the navy school in Rochefort was closed and partially replaced by the school in Le Havre, which existed only for one year. Finally, in 1786, the Gardes de la Marine were suppressed, and two non-military schools, namely the colleges in Alais and Vannes, were put in charge of the navy officers' instruction.

The artillery schools lasted longer than the navy schools. They were established in 1720 through a royal order, motivated by the success of some prototype schools like the one in Douai (that subsequently moved to Metz and then to Strasbourg, before being closed), and outlived until the French Revolution. Their history is closely related to that of the military engineers schools. Without any doubt, the most innovative mathematics teaching was delivered at the École du Génie in Mézières. From the beginning in 1748 on, an entry examination was instituted and the *académicien* Charles Camus was appointed *examinateur*. In 1756, the École du Génie was unified with the École

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d'Artillerie in La Fère and called École Royale des Élèves. This new institution was structured on the principles that had already inspired the École du Génie, and in particular, an entrance examination was set. Three years later, the two schools were split again. On the one hand, the École des Élèves was now reserved for artillery officers and was moved to Bapaume in 1765. It was closed in 1772.

On the other hand, the military engineer school was transfered back to Mézières and lasted until the Revolution. Moreover, a school (called the École Royale Militaire) was created in 1751 to train 500 nobles by birth who could not afford to pay for one of the existing schools. The best students were afterwards sent to the military engineers corp, the others to the artillery and, in the last instance, to the navy. In 1764, the Collège Henri IV in La Flèche, which previously belonged to the Jesuits, was merged with the École Royale Militaire and renamed École des Cadets; it was closed in 1776. Eleven military schools were founded in this year in order to fill this gap. With regard to these schools, Lacroix observed that they had been a "great experience that we did to refine the public teaching [*grande expérience que l'on fit pour perfectionner l'enseignement publique*]" (cf. Lacroix 1838, p. 50). We remark that the military school in Sorèze, previously a college run by the Benedectines, was especially renown for its pedagogical methods.

The list in Table 1 shows the most renowned military schools up to the French Revolution.

École Royale d'Hydrographie	1666	Le Havre
École d'Hydrographie	1673	Saint Malo
École d'Artillerie	1679	Douai
Écoles des Gardes du Pavillon et de la Marine	1689	Brest, Rochefort, Toulon
Écoles d'Artillerie	1720	Auxonne, Besançon, Grenoble, La Fère, Metz, Strasbourg, Valence
École du Genie	1748	Mézières
École Royale Militaire	1751	Paris
École Royale des Élèves	1756	La Fère
École Spéciale des Mineurs	1764	Verdun
École desCcadets	1764	La Flèche
École des Ponts et Chaussées	1747	Paris
École des Gardes du Pavillon et de la Marine	1773	Le Havre
École Royale de la Marine	1773	Le Havre

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Écoles Militaires	1776	Auxerre, Beaumont-en-Auge, Brienne, Effiat, Pont-à-Mousson, Pontlevoy, Rebais, Sorèze, Thiron, Tournon, Vendôme
École des Mines	1783	Paris

Table 1: List of military schools in France up to the French Revolution

With the French Revolution, the educational system underwent radical changes. For this reason, we take this event as upper bound and details about the further development after this period will not be thoroughly covered. In short, the *collèges* were transformed into the Écoles Centrales, which lasted until 1802. In 1794, the newly founded École Centrale des Travaux Publics was suppose to replace all the military engineers schools. Actually, one year later the school was renamed École Polytechnique and provided the general and theoretical training necessary for entering the Écoles d'Applications, which originated from the unification of the artillery and military engineers schools. Additionally, the schools for navy officers were definitively closed and replaced in 1791 by the Écoles d'Hydrographie et de Mathématiques, founded in the most important harbor towns. These schools were meant for the working-class, while the officers were educated on two training ships in Brest and Toulon, called the Écoles Spéciales de la Marine.

Finally, a few words on the institutions that supported the scientific research. During the Renaissance, the center of gravity of the intellectual life gradually passed from the universities to another kind of institution. These were the academies, which developed everywhere in Europe during the 17th and 18th centuries (for instance, in Rome in 1603, in Florence in 1657, in London in 1645, in Paris in 1666, in Berlin in 1700, in Moscow in 1725, and in Stockholm in 1769). In particular, at the *Académie des Sciences* in Paris, the scientific dialogue was conducted through epistolary exchanges and public debates and conferences. The *Académie* provided its members with rooms, laboratories, a library, and funds for experiences and missions. The members got together weekly to evaluate the scientific value of articles, books, and new inventions, in the first place of the members themselves. At the end of the 17th century, the *Académie* had considerably grown: it counted seventy members, organized into *honoraires*, *pensionnaires*, *associés*, *adjoints*, one secretary, and one treasurer. In addition to these, 85 *corréspondants*, who lived outside of Paris, were appointed. In 1793, the *Académie* was closed and finally restored in 1816.

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2.1.2 The Mathematical Sciences

Now, the question is: Which topics came within the mathematical sciences that were taught in these schools? With regard to the classification of knowledge, the two important categories of arts and sciences can be identified, for instance in the encyclopedias and, in particular, in the one by Diderot and d'Alembert. The arts were respectively divided into the mechanical ones, meaning the manual artisan crafts, and the liberal ones, for instance the languages, the art of war, and design. Medicine, law, and theology were instead located in the scope of the sciences and were sorted according to their increasing importance or "generality". Philosophy was considered propedeutical, inasmuch as it provided the conceptual tools that a student would need in his further studies. Even though the contents of philosophy studies often varied a lot, starting from the 17th century, we find that, to a greater or lesser extent, physics and mathematical sciences were also taught under the philosophy label.

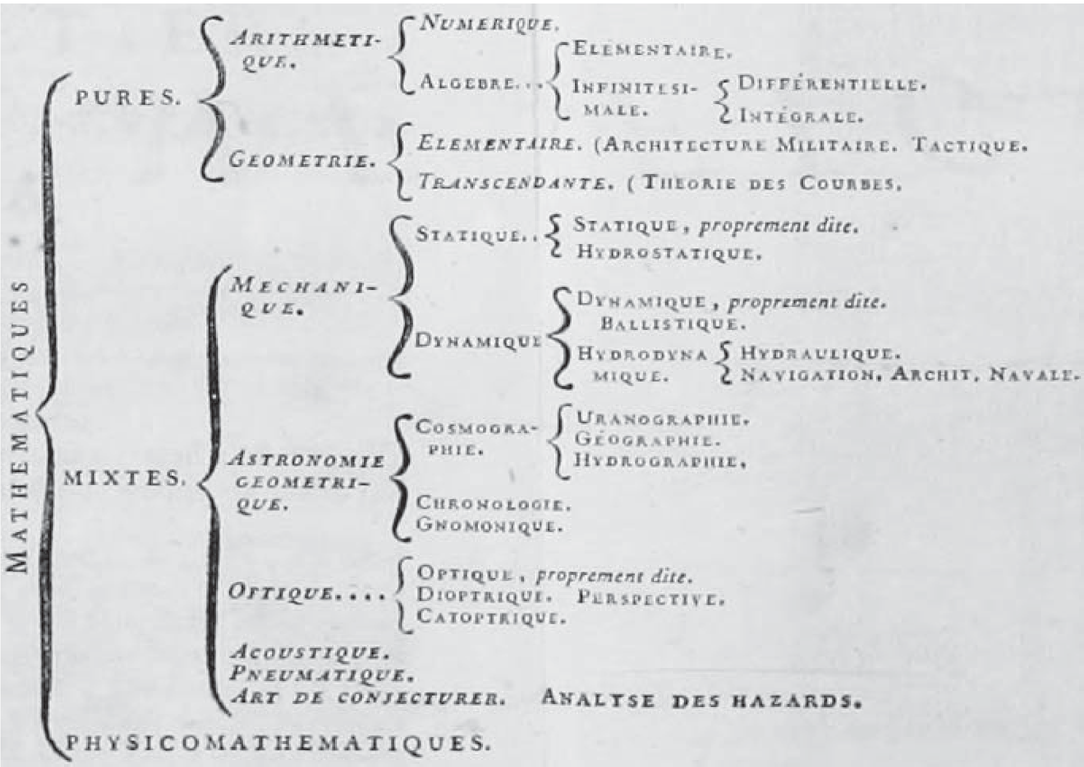

Illustration 1: Système figuré des connaissances humaines, cf. Diderot and d'Alembert, 1751-1765, vol. 1.

According to the *Encyclopédie* by Diderot and d'Alembert, mathematics is the science the object of which are the properties of magnitudes insofar as they are calculable or

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measurable.[3] Mathematics splits into two classes (cf. Illustration 1). The first one, called "pure mathematics", considers the properties of the magnitude in an abstract way and counts arithmetic (on calculable magnitudes or numbers) and geometry (on measurable magnitudes or extents). The second class, called "mixed mathematics", deals with the properties of concrete magnitudes, that is, magnitudes that are instantiated in some entities or specific subjects. It covers, for instance, mechanics, optics, astronomy, geography, chronology, hydrostatics, hydraulics, hydrography, and navigation (cf. Diderot and d'Alembert 1751-1765, vol. 10, pp. 188-189).

We remark that, surprisingly, military architecture and tactic are included in the pure mathematical sciences, since they are part of elementary geometry. Anyway, this was not the case for all the classifications of the mathematical sciences. For instance, Savérien in his *Dictionnaire Universel de Mathématique et de Physique* from 1753 (cf. Illustration 2) classified military architecture as an independent branch of mixed mathematics.

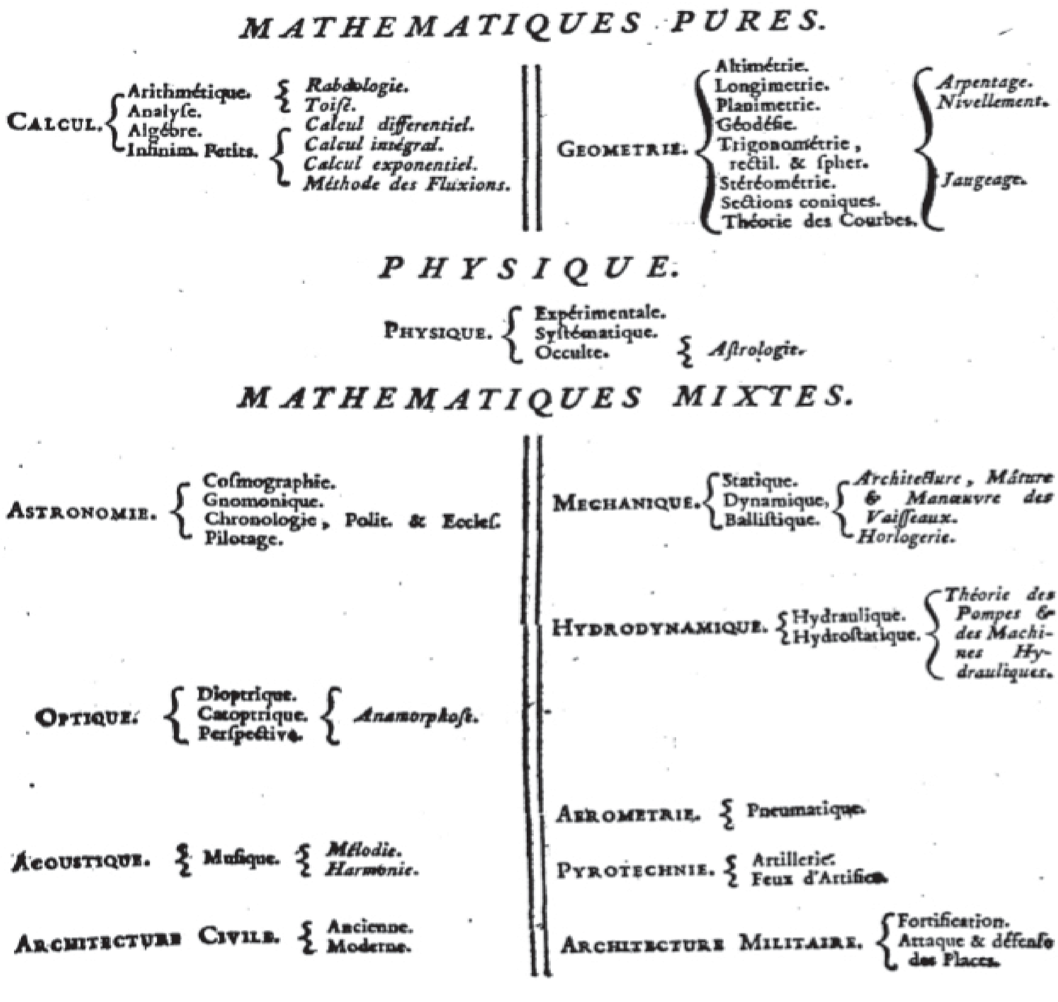

Illustration 2: Système figuré de sciences mathématiques, cf. Savérien 1753

3 "*Mathématique ou mathématiques c'est la science qui a pour objet les propriétés de la grandeur entant qu'elle est calculable ou mesurable*", cf. Diderot and d'Alembert, 1751-1765, vol. 10, p. 188.

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During the 18th century, the didactic concerns in the mathematical sciences underwent some changes. Since mathematics deals with natural entities in an abstract way it was considered, in the previous century, to be subordinated to physics, which deals with natural entities *tout court*. Therefore, it was taught after physics but as a matter of fact, due to the lack of time, it was usually not taught at all. In the 18th century, mathematics started to be considered as a real science that trains the intellect to argue. Moreover, thanks to the increasing mathematization of physics (especially of Newtonian physics), mathematics should be taught before physics; otherwise, the students would be lacking the far more sophisticated foundation that was required. We find *in nuce* the two main (sometimes opposed) arguments to justify the usefulness of studying mathematics: it is a mental training and it is needed in technical applications.

In practice, by the 1720s, the mathematical instruction that a student received often did not exceed the Euclidean principles. Afterwards, the situation changed substantially. If students had to understand a lecture on mathematical sciences like dynamics, statics, or hydrostatics, they needed at least the basic notions of algebra. We have few evidences to report on the period between the 1720s to the 1760s, but, at the end of this period, we know that even in smaller towns students were instructed in conic sections and most of them had also been introduced to calculus (cf. Brockliss 1987, p. 385). As already mentioned above, a common problem in teaching mathematics in this period was the lack of time. Only one year (the philosophy year) was provided to introduce the students to the basis in arithmetic, to calculus, and then to physics. Later on, this will lead to a splitting of the philosophy degrees.

2.2 General Description of the *Cours*

According to the *Encyclopédie* by Diderot and d'Alembert, a *cours* consists of "some elements and principles of a science, either written in a book or proved publically by experiments [*des élémens et des principes d'une science, ou rédigés par écrit dans un livre, ou démontrés en public par des expériences*]" (cf. Diderot and d'Alembert 1751-1765, vol. 4, p. 396). D'Alembert explained that this word apparently derives from the fact that one passes through (*parcourir*) all the topics that belong to the science at issue. The *cours* in a science should contain not only all the parts of this science and its principles, but also the most important details.[4] With regard to our topic, this implies that a written *cours*, or textbook, in the mathematical sciences is meant to be a complete course about these sciences.

4 "*Le mot de cours vient apparemment de ce qu'on y parcourt toutes les matieres qui appartiennent à la science qui en est l'object. Le cours d'une science doit contenir non seulement toutes les parties de cette science et leurs principes, mais les détails les plus importans*", cf. Diderot and d'Alembert, 1751-1765, vol.4, p. 396.

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Around the first half of the 18th century, the emergence of lots of new institutions, where the future officers were trained, led to the publication of new textbooks. Some pedagogical concerns inspired moreover the creation of new textbooks. They were needed in order to maximize the students' understanding by replacing the dictation practice during lectures. To this end, not only the teachers, but also the students were supposed to have them at hand.[5] This implies that, in the large, the textbooks were decreased in size, generally from *in folio* or *in quarto* to *in octavo*. For instance, Claude Françoit Milliet Dechales' textbook (1674) was an *in folio*, Jean Prestet's one (1675) and still the second edition of François Blondel's (1699) textbooks were *in quarto*, whereas La Caille's (1741) and Camus' (1749-1751) ones were *in octavo*. These textbooks usually evolved from series of lessons held by the authors and, in some cases, they had been ordered by an institution. In these textbooks one usually finds many solved examples and applications (more or less widespread according to each textbook, but it's a general tendency to pay attention to the applications). In general, there are no exercises to be solved by the students and *a fortiori* no solutions of these exercises.

In this period, the demand for new textbooks also depended on the fact that the mathematical sciences were gradually becoming a more important teaching subject. Indeed, in various kinds of military schools the mathematical sciences were a prominent topic. Therefore, their teaching underwent a revaluation. In addition to that, the older textbooks no longer complied with the latest technical requirements, especially after the reorganization of the French Royal Navy (1763) and the resultant demand for skilled officers.

As already mentioned in the , we have considered some criteria in order to identify the *corpus* of textbooks. Indeed, we have taken into account textbooks written in French for the teaching in higher education. Since, during the 18th century, students usually did not receive a mathematical basic knowledge before entering the higher education system, these textbooks start in most cases with elementary mathematics. The level of complexity that they reach is variable and depends on a lot of factors. These textbooks consist of one or more volumes. In most cases all have the same title, which usually is "*Cours de(s) mathématique(s)*" or "*Élémen(t)s de(s) mathématique(s)*", but in a few cases the volumes have different and specific titles. According to the inclusive classification of the mathematical sciences given above, these textbooks usually aim to provide a "complete" representation. Nevertheless, we have chosen to interpret "complete" in a loose way and, therefore, to include also textbooks in which mixed mathematics does not appear. Indeed, they were used in mathematics classes together with other

5 Of course, it is also likely that these new textbooks were used for self-teaching outside an institutional context, as it is sometimes written in the prefaces.

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textbooks on mixed mathematics, as it was the case for Clairaut's textbooks. These considerations led to the list of textbooks at page 113.

2.3 Authors

In the military schools and colleges, the teachers who taught the mathematical sciences were officially lecturing mathematics, and sometimes philosophy, physics, mechanics, or hydrography. The list in Table 2 shows the most famous teachers of the considered period who also wrote a textbook on mathematical sciences.

Bernard Forest de BÉLIDOR	École d'Artillerie (La Fère)	1720-1738
Claude-François BERTHELOT	École Royale Militaire	1760-1770
Etienne BÉZOUT	École des Élèves	1770-1772
François BLONDEL	Université de Paris	(?)
Charles BOSSUT	École du Génie (Mézières)	1766-1789
Charles CAMUS	Académie Royale d'Architecture	1730
Nicolas-Louis de LA CAILLE	Collège Mazarin (Paris)	1739-1762
Abbé DEIDIER	École d'Artillerie (La Fère)	1738-1746
Jean-Antoine DUCLOS	Collège de Lyon; Académie des Beaux-Arts	before 1737(?); before 1737(?)
Pierre HÉRIGONE	Paris	1634(?)
Johan Heinrich HERTTENSTEIN	École d'Artillerie (Strasbourg)	1720-1741
Paul HOSTE	École des Gardes de la Marine (Toulon)	1686-1700
Thomas Fantet de LAGNY	École d'Hydrographie (Rochefort)	1697
Bertrand LAMY	Collèges de Saumur, Angers; Seminaire de Grenoble	1669-1675; 1665-1687
Guillaume LEBLOND	École des Pages de la Grande Écurie	before 1747(?)
Jean-François MARIE	Collège Mazarin (Paris)	1770s
Roger MARTIN	Collège de Toulouse	1762
Jean-Antoine NOLLET	Collège Navarre (Paris)	1756-1770
Joseph PRIVAT DE MOLIÈRES	Collège de Saumur, Juilly, Soissons	1700(?)-1704
Jean PRESTET	Collèges de Nantes, Angers	1681
Jean SAURI	Université de Montpellier	before 1768
Dominique RIVARD	Collège Beauvais (Paris)	1735-1770
Pierre VARIGNON	Collège Mazarin (Paris)	1690

Table 2: French teachers who wrote a textbook on mathematical sciences

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Among the above listed professors, some were priests (Bossut, La Caille, Sauri) or took some orders, especially in the Oratorian Orders (Lamy, Privat de Molières). A good number of the above listed professors were members of the Académie des Sciences.

Another role of utmost importance for our research in this paper is the *examinateur*. This is in particular remarkable since one of the most popular authors of mathematical textbooks, Étienne Bézout, had never been a professor (except being assistant of Nollet for two years), but *examinateur* for the most parts of his life. This position was created to evaluate the students in the mathematical subjects, either to admit them to a military school, or to let them pass to the next class. The position of *examinateur* for the navy was filled by Bézout in 1764-1783 and then by Monge. The *examinateurs* of the artillery were: Camus up to 1768; then Bézout, who held this position concurrently with that in the navy until his death; and then Laplace from 1783 on. The *examinateurs* of the military engineers were: from 1756 on, Camus, who held this position concurrently with that in the navy until his death; and then Bossut from 1768.

Brief recalls of the biographies of the authors that we consider can be found in the (cf. pp. 97 ff.).

2.4 Time

We placed the starting point of our inquiry in 1634, when the first volume of the *Cursus Mathematicus* by Pierre Hérigone was published. Indeed, this is one of the first textbooks written in French – more precisely, it is a parallel Latin-French edition. Anyway, we focus mainly on 18th century textbooks. The endpoint of our inquiry is 1789, when the French Revolution brought along major changes, among others, in the educational system.

2.5 Contents and Structure of the Textbooks

In the following sections, we present a selection of the textbooks that are considered in detail. Out of about sixty works, we have chosen five, namely the textbooks by Bélidor (1725), La Caille (1741-1750), Camus (1749-1751), Bézout (1764-1769 and 1770-1772), and Bossut (1782). The benchmark is to consider, among the selection that we have identified according to the criteria displayed in the , the textbooks which, as far as we know, were most commonly used at that time. Moreover, they were all written during the 18th century in French with a teaching purpose and contain an inclusive presentation of the mathematical sciences, not only of pure mathematics.

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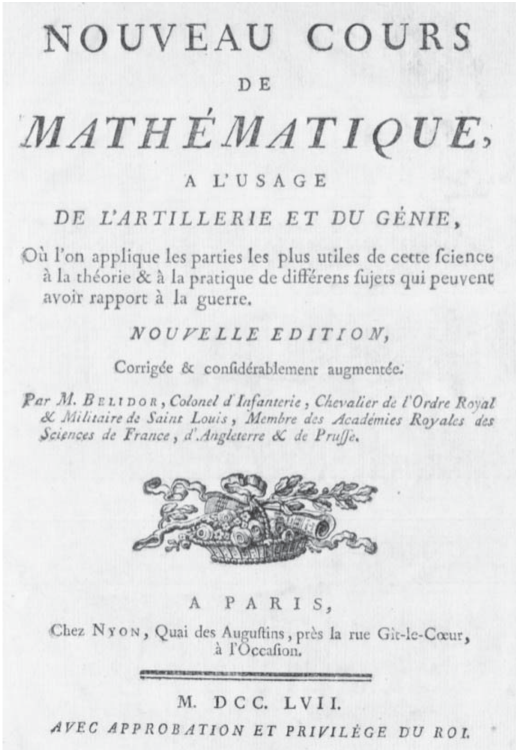
It seems that, from the first half of the 18th century on, the textbook by Camus provides an overview of the basics in the teaching of the mathematical sciences for the students of the army officer schools (cf. Taton 1964, p. 535). Camus' work is characterized by an elementary level. The textbooks by Bézout and Bossut begin to show a certain uniformization in contents, though they also present various differences. The textbooks by La Caille and Bélidor were also two classic readings, though a little earlier in time. Bélidor's textbook, in particular, is the most old-fashioned, both in the topics dealt with and in the way of presenting them. While all the other textbooks were used mainly in military schools, La Caille's was conceived for students in a *collège*.

2.5.1 The *Nouveau Cours de Mathématiques* (1725) by Bélidor


NOUVEAU COURS

DE

MATHÉMATIQUE,

A L'USAGE

DE L'ARTILLERIE ET DU GÉNIE,

Où l'on applique les parties les plus utiles de cette ſcience à la théorie & à la pratique de différens ſujets qui peuvent avoir rapport à la guerre.

NOUVELLE EDITION,

Corrigée & conſidérablement augmentée.

Par M. BELIDOR, *Colonel d'Infanterie, Chevalier de l'Ordre Royal & Militaire de Saint Louis, Membre des Académies Royales des Sciences de France, d'Angleterre & de Pruſſe.*

A PARIS,

Chez NYON, Quai des Auguſtins, près la rue Gît-le-Cœur, à l'Occaſion.

M. DCC. LVII.

AVEC APPROBATION ET PRIVILÉGE DU ROI.

Illustration 3: Title page of the Nouveau Cours de Mathématiques *(1725) by Bélidor*

Bélidor wrote several textbooks for civil and military engineers. His work *Nouveau Cours de Mathématiques à l'Usage de l'Artillerie et du Génie* was spread as a manuscript version since 1722, and, in 1725, was published in a *in folio* volume. This textbook built Belidor's reputation. It was originally conceived for the artillery students in La Fère, where Bélidor was a professor, but it was commonly used in artillery schools and also at the École Royale des Ponts et Chaussées. For almost two decades, this textbook, together with the *Mémoires d'Artillerie* by Surirey de Saint-Rémy and with the *Elémens Généraux* by Deidier, constituted the knowledge demanded of an artillery

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officer. Bélidor also wrote other books for the military corps, which all had several editions. Among them, the *La Science des Ingénieurs* (1729) deals with fortifications, the *Le Bombardier François* (1731) was for the use in combat, and the *Architecture Hydraulique* (1737-1739) embraced civil constructions. As Grandjean de Fourcy remembers in his eulogy, the fact that the topics treated in Bélidor's *Nouveau Cours* were considered the overall knowledge that an officer needed led him to be acknowledged as a "general professor" of mathematics (cf. Fourcy 1761, p. 171).

The *Nouveau Cours* consists of sixteen parts, called "*livres*". Half of them (Part I to VIII) are devoted to pure mathematics, namely arithmetic, geometry, and algebra. After that, we find conic sections and their application to projectiles trajectories, linear trigonometry and leveling, measuring by using the *toisé* unit measure and how to construct frameworks for buildings, the measures of regular and irregular surfaces and solids, how to divide fields and how to use a sector, how to deal with alloys, the study of moving bodies and bomb throwing, and finally static mechanics, hydrostatic, and hydraulic. Compared to the textbooks that we take into account in the following, Bélidor's is the most ancient. The topics are organized in fragmented series of heterogeneous subjects, such as, among others, the usage of the old unity of measure of the *toisé*. The discussion is elementary and, in the study of the mathematical sciences, mathematics is used in a still superficial way. This textbook was replaced in 1738 by Deidier's because of political reasons (cf. Taton 1964, p. 528).

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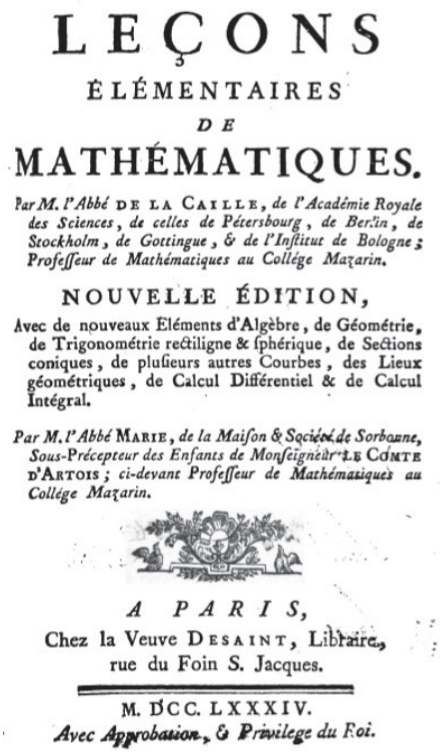
2.5.2 The *Leçons Elémentaires* (1741-1750) by La Caille



LEÇONS
ÉLÉMENTAIRES
DE
MATHÉMATIQUES.

Par M. l'Abbé DE LA CAILLE, de l'Académie Royale des Sciences, de celles de Pétersbourg, de Berlin, de Stockholm, de Gottingue, & de l'Institut de Bologne; Professeur de Mathématiques au Collége Mazarin.

NOUVELLE ÉDITION,

Avec de nouveaux Eléments d'Algèbre, de Géométrie, de Trigonométrie rectiligne & sphérique, de Sections coniques, de plusieurs autres Courbes, des Lieux géométriques, de Calcul Différentiel & de Calcul Intégral.

Par M. l'Abbé MARIE, de la Maison & Société de Sorbonne, Sous-Précepteur des Enfants de Monseigneur LE COMTE D'ARTOIS; ci-devant Professeur de Mathématiques au Collége Mazarin.

A PARIS,
Chez la Veuve DESAINT, Libraire, rue du Foin S. Jacques.

M. DCC. LXXXIV.
Avec Approbation, & Privilege du Roi.

Illustration 4: Title page of the first volume of the Leçons Elémentaires *(1784, second edition) by La Caille*

While La Caille was in charge of the mathematics teaching at the Collège Mazarin, he gave permission to let his lectures be printed. The outcome is a complete *in octavo* textbook about the mathematical sciences, which consists of four volumes. The first volume was promptly translated into Latin, Spanish, and English, and then the other volumes followed. The textbook was extended by Marie in 1770 and republished until 1811. It was reasonably supposed to replace the textbook by Jean-Mathurin Mazéas (1758), which lacked in differential and integral calculus and was no more published in Paris after 1776.

The *Leçons Elémentaires de Mathématiques* deal with arithmetic, algebra, geometry, conic sections, and (less than two decades after Bélidor's textbook) with differential and integral calculus. Except for this last topic, the contents of this textbook are similar to Varignon's. The *Leçons Elémentaires d'Astronomie* had been firstly published in 1743 and then extended in 1779 by Jerôme Lalande, who was a former student of La Caille. They represent the knowledge that La Caille had acquired since he practiced astronomy as a profession. The *Leçons Elémentaires d'Optique* and the *Leçons Elémentaires de Mécanique* were published in 1746 and 1750, respectively. The for-

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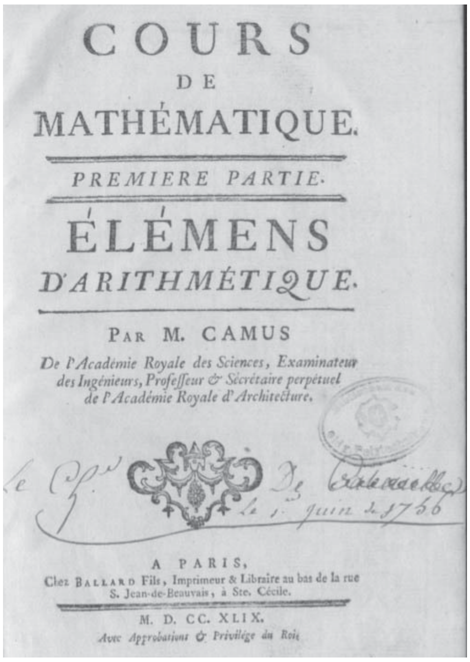
mer deals with optics, catoptrics, dioptrics, and perspective, which were all quite common subjects at that time. Any description of instruments or machines is left out because La Caille rather regarded them as belonging to experimental physics. The lecture on mechanics, instead, should not focus on the machines, but on everything that can be moved and that can move something else, that is, on the whole matter. With regard to his reputation as astronomer, La Caille explained that the creation of his textbook about mechanics resulted from his dissatisfaction with the existing books on the topic. This deals with linear motion, shock, and circular motion, trying to reduce their principles to a clear and methodical system.

2.5.3 The *Cours de Mathématiques* (1749-1751) by Camus



COURS
DE
MATHÉMATIQUE.
PREMIERE PARTIE.
ÉLÉMENS
D'ARITHMÉTIQUE.
PAR M. CAMUS
De l'Académie Royale des Sciences, Examinateur des Ingénieurs, Professeur & Sécrétaire perpétuel de l'Académie Royale d'Architecture.
A PARIS,
Chez BALLARD Fils, Imprimeur & Libraire au bas de la rue S. Jean-de-Beauvais, à Ste. Cécile.
M. D. CC. XLIX.
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Illustration 5: Title page of the first volume of the Cours de mathématiques *(1749) by Camus*

Camus' textbook is based on the lectures that he delivered at the Académie Royale d'Architecture, where he was appointed professor in 1730. The textbook was ordered by the Minister of War the Comte d'Argenson, who also determined the topics to be included, for the military engineers at Mézières. As a consequence (and as it will be later the case for Bézout), Camus' textbook easily received the governmental *imprimatur*. When the corps of the military engineers and the artillery were merged in 1756, Camus' textbook was also used in the artillery schools. Considering that Camus became the examiner for both schools, it is not surprising that the book achieved large success (four complete editions up to 1769).

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Camus' *Cours de Mathématiques* is divided into three volumes. The first one, which dates back to 1749, deals with arithmetics. Its main topics are numbers, proportions, alloys, progressions, and combinations. The second volume, published in 1750, is about geometry, namely about lines, surfaces, proportions, solids, plane trigonometry, and some special curves. The last volume from 1751 treats mechanics, and more precisely statics. As already mentioned above, it deals with gravity centers, but also – even if to a far less extent – with forces and machines. According to d'Argenson's instructions, there should have been a fourth volume on hydraulics, but it had never been written. The *Traité Elémentaire de Mécanique et Dynamique* (1763) by Bossut was intended to complete Camus' *Cours*. Moreover, among the number of manuscripts left at Camus' death, a work on hydraulics was found, apparently intended to complete the *Cours*. There should have also been another volume which should have combined analysis and algebra (intended as calculations with letters or "*calcul littéral*"), but it has never been published. We could interpret the following argument by Camus as an explanation: he claimed that calculations with letters should not be treated together with numerical ones (that is, arithmetic), since this would not agree with the mandatory topics that engineers had to be instructed in (cf. Camus 1749, p. iv). This textbook, as the one by La Caille, reveals a significant change of topics compared to Bélidor's. Especially, the range of the mathematical sciences is getting narrower and narrower, and, at the same time, more technically specialized, including calculus and a detailed part on mechanics.

Camus' textbook was not exempt from criticism. On the one hand, it was strongly disapproved sinceit was considered inappropriate for the artillery. Indeed, we recall that it had been conceived for the military engineers, but adopted in the artillery schools without any change. We remark, in particular, that more than two-thirds of the volume on mechanics is devoted to gravity centers, and only a minimal part deals with forces. In 1762, Berthelot published the first volume (and unique, according to Taton 1964, p. 534) of a more basic textbook that should have replaced Camus' for the artillery. On the other hand, the textbook was accused of being too elementary for the military engineers at Mézières. Already fifteen years after its composition it was judged no more up to date: in 1764, Le Cozic remarked that it fell short in analysis, dynamics, hydrostatics, and hydraulics, and that it included few practical applications. Furthermore, compared to contemporary textbooks, conic sections were alsomissing. In the end, Camus' textbook was replaced by Bézout's.

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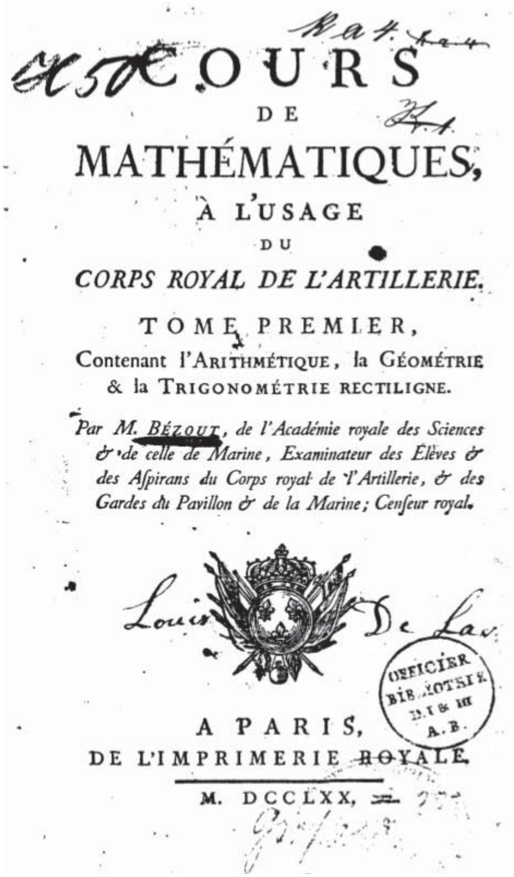
2.5.4 The *Cours de Mathématiques à l'Usage des Gardes du Pavillon et de la Marine* (1764-1769) and the *Cours de Mathématiques à l'Usage du Corps Royal de l'Artillerie* (1770-1772) by Bézout



COURS
DE
MATHÉMATIQUES,
À L'USAGE
DU
CORPS ROYAL DE L'ARTILLERIE.
TOME PREMIER,
Contenant l'Arithmétique, la Géométrie & la Trigonométrie rectiligne.
Par M. Bézout, de l'Académie royale des Sciences & de celle de Marine, Examinateur des Élèves & des Aspirans du Corps royal de l'Artillerie, & des Gardes du Pavillon & de la Marine; Censeur royal.
A PARIS,
DE L'IMPRIMERIE ROYALE.
M. DCCLXX.

Illustration 6: Title page of the first volume of the Cours de Mathématiques à l'Usage du Corps Royal de l'Artillerie *(1770) by Bézout*

Bézout wrote two *Cours*, both commissioned by the Duc de Choiseul. The first had been intended for the navy and was written when Bézout was appointed *examinateur* of this corp. The second was conceived to replace Camus' textbook for the artillery since the criticisms against it could no longer be ignored.

These textbooks are practically oriented because they were intended to instruct students in the elementary mathematical sciences that a navy or artillery officer needed. The *Cours* for the navy consists of six volumes, respectively dealing with: arithmetic; geometry and linear and spherical trigonometry; algebra and its applications to arithmetic and geometry; differential and integral calculus, principles of mechanics, and equilibrium of fluids; statics and dynamics, machines, and ballistic; and finally navigation. The fact that algebra is treated after geometry fits Bézout's didactic requirements,

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since he observed that beginners were not yet familiar with mathematical reasoning to understand the force of algebraic demonstrations, although they did appreciate proofs in geometry. Moreover, the conic sections are presented together with mechanics, which were, instead, missing in Camus' textbook. The *Cours* for the artillery consists of four volumes, respectively dealing with: arithmetic, geometry, and linear trigonometry; algebra and its applications to arithmetic and geometry; differential and integral calculus principles of mechanics, and equilibrium of fluids; and finally statics and dynamics, machines, and ballistics. Bézout planned a fifth volume, the equivalent of the navigation volume in the navy textbook, but his project was prevented by the closing of the Ecole des Élèves for the artillery in 1772.

Since the artillery textbook is mainly patterned as a shorter version of the navy textbook, the differences between the two textbooks are narrow. At first sight, we notice that the topics of the first three volumes of the navy textbook are covered in only two volumes of the artillery textbook. This latter contains fewer propositions, but the previous numbering from the navy textbook is preserved, which produces gaps in the numeration whenever a proposition from the navy textbook had been skipped. The same relation exists between the fourth volume of the navy textbook and the third volume of the artillery textbook, but the propositions' numbering is not reproduced. We moreover notice that the navy textbook contains a whole volume on navigation – which obviously is not part of the artillery textbook. The relation between the fifth volume of the navy textbook and the fourth volume of the artillery textbook is more entangled. Like the previous volumes, they both follow the same general structure, but in some cases the artillery textbook is more comprehensive. Especially in the case of topics concerning ballistics, parts of them are explained on an advanced level in one textbook compared to the other, and vice versa for other parts. Summing up, the artillery textbook is overall a refitted and simplified version of the *Cours* for the navy, where the same essential structure and the numerous applications are maintained. The level in algebra is lower, Bézout's researches did not appear anymore, and the small characters point now to the consequences of the statements printed in normal characters, instead of being, as it was the case in the navy textbook, the most in-depth parts for advanced students. Nevertheless, Bézout maintained a high level in the last volume on ballistics.

Due to their similarities, the two works were merged in 1795 by Peyrard under the title *Cours de Mathématiques à l'Usage de la Marine et de l'Artillerie*. It was used by students preparing for the aptitude test to enter the École Polytechnique. Bézout's textbooks were widely used in France over the years. Nevertheless, they were also criticized: indeed, because of his approach, Bézout was occasionally reproached with a

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lack of rigor. His textbooks were so successful that they were reprinted until the 19^{th} century and translated into many languages. They had up to 21 editions and were revised, among others, by Peyrard, Garnier, Reynaud, Lacroix, Saigey, and Honoré. On a side note, we still find some traces of Bézout's textbooks in several autobiographical writings. For instance, in 1785, Napoleon Bonaparte wrote in his copy of the textbook: "*Grand Bezout, achève ton cours, /Mais avant permets-moi de dire/Qu'aux aspirants tu donnes secours./Cela est parfaitement vrai,/Mais je ne cesserai pas de rire/Lorsque je l'aurai achevé/Pour le plus tard au mois de mai;/Je ferai alors le conseiller*". Also Stendhal recalled in the *Vie de Henry Brulard* that, as he was attending the École Centrale in Grenoble around 1796, "*Nous suivions le plat cours de Bezout mais M. Dupuy eut le bon esprit de nous parler de Clairaut et de la nouvelle édition que M. Biot (ce charlatan travailleur) venait d'en donner. Clairaut était fait pour ouvrir l'esprit que Bezout tendait à laisser à jamais bouché. Chaque proposition dans Bezout a l'air d'un grand secret appris d'une bonne femme voisine*". Even Jean Baptiste Joseph Fourier studied Bézout's textbook around 1780 (cf. The MacTutor History of Mathematics Archive online at http://www-history.mcs.st-and.ac.uk/).

Nowadays Bézout's name is still well-known in contemporary mathematics, mainly due to two theorems. The first one, known as Bézout's theorem (in French: *théorème de Bézout*), is a statement of algebraic geometry according to which the number of common points of two plane algebraic curves is equal to the product of their degrees. This is the geometrical version of a theorem about the degree of the resultant of n equations in n unknowns, that Bézout proved at first for $n = 2$ in 1764 and for any n in 1779. The second one, known as Bézout's identity or Bézout's lemma, is a statement of the elementary number theory. It says that, taken a and b nonzero integers and let d be their greatest common divisor, then there exist integers x and y such that $ax + by = d$. Moreover, if a and b are coprime, then $ax + by = 1$. Such a formulation for integers was firstly stated by Claude Bachet de Méziriac in 1624, and indeed the first part of the theorem is called in French *identité de Bézout* or *théorème de Bachet-Bézout*, while the second part is (again) the *théorème de Bézout*. In truth, in a research article from 1764, Bézout proved a more general version for polynomials, namely that two polynomials in one variable have a common root if and only if their resultant is 0. Today's version is equivalent to it and says that two polynomials $P(x)$ and $Q(x)$ with coefficients in a field K are coprime if and only if there exist two polynomials $U(x)$ and $V(x)$ such that $U(x)P(x) + V(x)Q(x) = 1$. If we ask the degree of $U(x)$ to be less than the degree of $Q(x)$ and the degree of $V(x)$ to be less than the degree of $P(x)$, the couple (U, V) is unique. Until the emergence of the notion of an ideal on a ring, this theorem was not

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considered of as particular important. At first Georges Papelier in 1903, and then Bourbaki in 1948, associated again this theorem with Bézout's name.

2.5.5 The *Cours de Mathématiques* (1771-1775) by Bossut

TRAITÉ
ÉLÉMENTAIRE
D'ARITHMÉTIQUE:
Par M. l'Abbé BOSSUT, de l'Académie Royale des Sciences, Examinateur des Ingénieurs, &c.

A PARIS,
Chez CLAUDE-ANTOINE JOMBERT, Fils aîné, Libraire, rue Dauphine, près le Pont-Neuf.

M. DCC. LXXII.
Avec Approbation & Privilége du Roi.

Illustration 7: Title page of the Traité Elémentaire d'Arithmétique *(1772) by Bossut*

Bossut wrote a series of textbooks that appeared in a lot of French and foreign-language editions. Five of them, the *Traité Elémentaire d'Arithmétique* (1772), the *Traité Elémentaire d'Algèbre* (1772), the *Traité Elémentaire de Géométrie* (1773), the *Traité Elémentaire de Mécanique* (1775), the *Traité Elémentaire d'Hydrodynamique* (1771) in two volumes formed the *Cours de Mathématiques* for the École du Génie, together with the French translations of the second volume of Agnesi's *Traités Elémentaires de Calcul Différentiel et de Calcul Intégral* (1775). The *Cours* was commissioned by the Duc de Choiseuil. On the cover page of all these textbooks, the title explicitly says "Cours de Mathématiques" and the number of the volume that each textbook were supposed to cover, except the hydrodynamics one and the translation from Agnesi, is stated. Nevertheless, each volume has to be considered as a unique textbook (cf. Taton 1964, p. 584). Indeed, in 1800, seven volumes were published under the unitary title "Cours de Mathématiques": the volumes correspond to the above treatises, with the slight exceptions that arithmetic and algebra are jointed together, and that there are two volumes about differential and integral calculus now.

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We recall that Bossut had already completed Camus' textbook with the *Traité Elémentaire de Mécanique et Dyn*amique (1763) and with the *Traité Elémentaire d'Hydrodyn*amique (1771), the latter being also part of his own textbook. Bossut tried to modernize the presentation of the lectures at the École du Génie, but he didn't achieved it before Camus' death. Afterwards, Bossut's textbook was intended to replace, for the Génie, Camus'. Bossut was also the author of the *Essai sur l'Histoire Générale des Mathématiques*, that inspired the introductions to the volumes of the 1800 *Cours*.

The *Traité Elémentaire d'Arithmétique* deals with the same topics as the first volume of Camus' *Cours*, with the addition of logarithms. The *Traité Elémentaire d'Algèbre* deals at first with operations with algebraical quantities. Afterwards, we find the solutions by radicals to equations of the 2^{nd} to the 4^{th} degree, some methods concerning the equations in general (about equal roots, approximations, and eliminations), and sequences. The *Traité Elémentaire de Géométrie* deals with lines, surfaces, solids, trigonometry, and, in addition to Camus' textbook, with applications of algebra to geometry, that is, with the study of functions. The *Traité Elémentaire de Mécanique* deals with statics and dynamics, and in particular with the gravity centers and machines. The first volume of the *Traité Elémentaire d'Hydrodynamique* deals with hydrostatics and hydraulics, while the second covers hydrodynamics. Finally, the translation from Agnesi's *Traités Elémentaires de Calcul Différentiel et de Calcul Intégral* also includes the inverse tangent method.

The *Cours* by Bossut, together with Bézout's, represented the emergence of a standardized, rigorous system of textbooks in the mathematical sciences during the 18^{th} century. Bossut's textbook was used, besides at the École du Génie, also at the Benedictine Collège in Sorèze, at the Collège de France, at the École des Ponts et Chaussées, and at the École des Mines.

2.6 Intention and Addressed Public

In the considered period, new textbooks were published due to a renewed demand. As was already mentioned, these constituted the main introductory textbooks since they were used for the first mathematical instruction that the students received. Therefore, they incorporated basic knowledge as well as (in some cases) less elementary subjects. In general, they sufficed the new pedagogical requirements.

It is likely that also students were supposed to buy these now small (*in octavo* or smaller) textbooks. Unfortunately, due to the lack of recordings, it is extremely difficult to estimate the number of students in the military schools and colleges, and to pre-

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cisely identify their background or their professional orientation. Only few can thus be concluded concerning the addressed public of the mathematical textbooks of this period. Moreover, it is possible that also self-taught learners used them. Concerning the selection that we consider – if we want to set a rough classification – the textbooks by Camus and Bossut were mainly used for the instruction of military engineers, Bélidor (and Deidier) for the artillery, and Bézout for the navy and, later, for the artillery. La Caille's textbook was used at the Collège Mazarin, where he was professor, and in some Benedectine colleges (especially the volume on astronomy).

Information on the didactic orientation of a textbook can usually be found in the prefaces. As Bélidor wrote, he believed that arithmetic, algebra, and geometry are the common base of all mixed mathematics, but he also believed that officers and military engineers should not study mathematics too thoroughly, like those who want to dedicate their whole life to it. Thus, he made an effort to gather everything that a military engineer needed to know in one volume with examples and applications. Bélidor indicated some pedagogical concerns, mainly focused on helping beginners. On the one hand, he wanted an engineer to know the reasons of the results that he uses, since one performs operations more surely being aware, for instance, of the nature of the employed numbers. On the other hand, he tried to simplify the exposition by shortening the operations as much as possible.

In his prefaces, La Caille also presented a number of pedagogical concerns. First of all, he felt the need to remark that French was more suitable than Latin to explain the mathematical sciences. Moreover, he argued that teaching mathematical sciences in Latin was an ancient, almost abandoned method since the language barrier did not foster the students' understanding of the mathematical contents (cf. La Caille 1766, p. 22). It is well-known that textbooks in French existed since the 16th century. Thus, this could attest La Caille's will to distance himself from the Jesuit tradition in colleges and universities where Latin texts still might have been used. Going back to the pedagogical concerns, La Caille tried to "squeeze a bit" the mathematical contents in the first volume in order to reach a double advantage. Indeed, on the one hand, he managed to deal briefly with pure mathematics, while, on the other hand, he tried to make the reader active by omitting too detailed explanations. Since the textbook contains such brief accounts, explications by a teacher were necessary to benefit from it – La Caille himself said that the oral presentation is its core ("*âme*"). Actually, this textbook should rather be considered as a printed exercise book, so that students and teachers could save time by avoiding transcribing lectures. As it will be also the case in Bézout's textbooks, La Caille used two different character sizes, while the smaller one

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is intended for advanced topics. As a general rule, he preferred indirect methods, as the double false position, even if they were not as elegant as geometers preferred. Indeed, they have a valuable advantage over the direct methods, since, in most of the cases, they are easier to apply in practice and lead to less cumbersome calculations.

Camus' pedagogical positions radically differ from the preceding ones: he did not seem to be concerned with simplifying tasks for beginners. According to the prefaces of his textbooks, Camus' aim was rather to develop the topics in a detailed way since he believed the existing textbooks to be superficial. On the polar opposite of Alexis Clairaut, who, between 1741 and 1746, wrote some textbooks, in which he praised the pedagogical value of letting the reader gradually discover abstract propositions starting from particular problems, Camus' praised the "synthetic" or inductive way. Therefore, he opened his textbook by explaining precisely the meaning of "definition", "theorem", "problem", "corollary", "remark", and "scholium". His opinion had been strongly criticized by his contemporaries, who blamed him for having written a book too elaborated for beginners. Even Grandjean de Fourcy cannot avoid to recall in his eulogy on Camus that such an inductive method makes the book more difficult and laborious for beginners to read (cf. Fourcy 1768, p. 152).

In the preface to the *Cours* for the navy, Bézout claimed that he wanted to provide the elementary mathematical knowledge that a future officer needed. He was though aware that his readership was formed by non-mathematicians and, therefore, avoided technical terms like "axiom" or "theorem" and too detailed arguments. His intention was tempered by the time limits of usual curricula in military schools: he limited himself to the mathematical topics useful for navigation and, at the same time, included numerous examples in order to make the study easier. Nevertheless, also advanced topics are contained in his textbooks: these are printed in smaller characters and designated for the best students. Bézout recognized the beginners' need to generalize ideas in order to fully appropriate the contents: he avoided to multiply methods and techniques regarding one subject to keep the beginner focused on it. Anyway, he still tried to present a topic from several viewpoints since beginners also needed a certain amount of knowledge. Bézout claimed to pay attention to the language and was concerned with keeping it simple.

Contrary to his predecessors, Bossut did not explicitly formulate his didactic intentions in the prefaces of his *Cours*. If a preface ("*Discours préliminaire*") is added to one of the volumes, it usually has a historical character and mainly recalls Bossut's *Essai sur l'Histoire Générale des Mathématiques*. Some information can be found in the dedication of the second and fifth volume. There, Bossut recalled that the Duc de Choiseuil

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commissioned a textbook for the Ecole du Génie in which theory and praxis should have been equally integrated.

2.7 Usage and Dissemination

During the 18th century, the changes in the structure of French mathematical textbooks went into the direction of a more technical presentation and, at the same time, no longer gave a highly comprehensive picture of the mathematical sciences. This turn is for sure linked to the changes in the educational system, and, especially, to the creation of military schools to improve the army standard. Indeed, at that time, the mathematical sciences were taught in-depth in these schools, thus becoming a prominent teaching subject.

Bélidor's textbook was firstly used by artillery students in La Fère, then commonly in all artillery schools and at the École Royale des Ponts et Chaussées. In contrast, La Caille's was written for the Collège Mazarin. It had been translated into many languages, and was republished until the 19th century. Camus' textbook had been commissioned for military engineers at Mézières. As such, it received the governmental *imprimatur*, but it had also been highly criticized. Bézout's textbooks were commissioned firstly for the navy, then for the artillery. They had been merged together, translated, and were republished until the 19th century. Finally, Bossut's textbook were conceived for military engineers. They had also been translated and experienced several editions.

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3 The *Anfangsgründe* Literature

3.1 Educational Circumstances in Germany during the 18th Century

In order to embed the *Anfangsgründe* it is necessary to give a short overview of the German educational system, especially of the universities in the 18th century. In the case of Germany, it is not possible to give a uniform overview, because the country – and along with that also the educational system – was territorially and confessionally split. There was a distinction between catholic and protestant territories and universities. The Jesuit Order was mainly responsible for the education at catholic universities until its dissolution in 1773. The Jesuits were said to be behindhand because they followed their curriculum "*ratio studiorum*" from 1599 (cf. Rüegg 1996, p. 112). The protestant universities were coined by the Enlightenment. There was a change of teaching methods: one distanced from rote memorization and spoke for learning through insights. The philosophers of the Enlightenment also wanted to establish German as scientific language. Already in the 17th century, Christian Thomasius (1655-1728) introduced the German language at university and lectured in German at the University of Leipzig (cf. Kühn 1987, p. 15). Over time, new German textbooks were needed. This development can mainly be noticed at protestant universities.

It is important to notice that there was no uniform compulsory education in 18th century Germany. One could enroll at a university without having attended a public school before.[6] In order to guarantee a unified knowledge level, every student had to pass a propaedeutic study within the arts faculty. This was the so called lower faculty in contrast to the three higher faculties medicine, law, and theology. Mathematics was also part of this propaedeutic study and no independent discipline. Nevertheless, efforts were made to establish mathematics as an independent subject. The first step to this direction was the publication of extensive textbooks like the *Anfangsgründe* which covered all disciplines belonging to the mathematical sciences in the 18th century. Thereby, the authors could show the extent and the utility of the mathematical sciences in everyday life.

6 In 1788, Prussia regulated the university entrance qualification with the "*Abitur*" certificate (cf. Geißler 2011, p. 82).

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3.2 General Description of the *Anfangsgründe*

It is difficult to find an exact description of the *Anfangsgründe*. This is the reason why we target to extract some characteristics of these textbooks. For this purpose, we studied various mathematical *Anfangsgründe*, especially the prefaces in which the authors described their approaches and their aims. The following results of our research might contain overlaps because the particular aspects are interdependent.

Today, the term *Anfangsgründe* is not common in Germany. In 18^{th} century, this term was used for textbooks which introduced a topic. *Anfangsgründe* can be translated with “elements” or “basics”. From this, one can infer that basic mathematical elements were presented in these textbooks. The expression was not only used for mathematical textbooks, but also for textbooks of other disciplines. Thus, the name was not bound to a specific topic. The authors of the mathematical *Anfangsgründe* which we focused on, did not explain this term. This is a hint that it was common during this period.

The mathematical *Anfangsgründe* can be described as introductory, scientific textbooks which were created to assist the mathematical sciences teaching at German universities. In Germany, as we have seen in France, students usually did not have any mathematical training before entering university. This is the reason why elementary contents were integrated in the *Anfangsgründe*. For instance, at the beginning of the arithmetic chapter, the authors describe how to write a number. Then they proceed to higher knowledge gradually. In some textbooks, higher mathematics is included in the form of calculus and algebra. One can even find some remarks on mathematical research problems. These topics were beyond the scope of the regular syllabus at universities, where the focus was put on the imparting of basic knowledge within the propaedeutic study. Consequently, the *Anfangsgründe* can be regarded as scientific textbooks.

The peculiar aspect of the *Anfangsgründe* is the usage of the German language. Before the 18^{th} century, Latin was the scientific language and textbooks for higher education were written in Latin. The *Anfangsgründe* are the first textbooks written in German for the use at universities. This shift goes along with one of the aims of the Enlightenment movement, which was the establishment of German as a scientific language. Thereby, it was possible to reach a broader audience, namely also those people who did not know Latin. During the 18^{th} century, the number textbooks in German on the mathematical sciences increased dramatically (cf. Wagner 1985, pp. 115 ff.).

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3.3 Authors

In our research, we focused mainly on six authors: Christian Wolff (1679-1754), Abraham Gotthelf Kästner (1719-1800), Johann Andreas von Segner (1704-1777), Heinrich Wilhelm Clemm (1726-1775), Wenceslaus Johann Gustav Karsten (1732-1787), and Georg Simon Klügel (1739-1812). We chose their textbooks because they were the most popular in the 18th century and the most cited in secondary literature. Another reason for our choice is that these textbooks contain almost all disciplines which were counted among the mathematical sciences during this time.

Except for Clemm, the mentioned authors were all professors of mathematics (and physics) at a German protestant university. Wolff was professor at the universities of Halle and Marburg. Kästner was professor at the University of Göttingen. He was the successor of Segner, who was first at the University of Göttingen, later at the University of Halle as the successor of Wolff. Karsten was at the universities of Rostock, Bützow, and Halle. Klügel, one of Kästner's students, became professor at the universities of Helmstedt and Halle. Clemm is the only one who did not have a teaching position at a university, but at a grammar school (*Gymnasium*) in Stuttgart.

The first person who published some mathematical *Anfangsgründe* was Wolff. This can be seen as the starting point of the so-called *Anfangsgründe* tradition. Wolff published his four-volume *Anfangsgründe aller mathematischen Wissenschaften* in 1710. They were so popular and frequently used that they were reprinted until 1800, a long time after Wolff's death. Until the second half of the 18th century, Wolff's textbooks were without any competition. Then, the next generation of authors of *Anfangsgründe* published their own textbooks. The most comprehensive one is Kästner's *Mathematische Anfangsgründe*. He started his textbook as a six-volume creation in 1758. Over the years, some volumes were added, so that the *Anfangsgründe* ended up having ten volumes. Some of the volumes were reprinted, for example the first one with the title *Anfangsgründe der Arithmetik, Geometrie, ebenen und sphärischen Trigonometrie, und Perspectiv*, the sixth edition of which was published in 1800. Because of the popularity of Kästner's textbook, we pay special attention to it. For this purpose, we take a closer look on Kästner as mathematical teacher and textbook-writer. Thanks to the comprehensive biographical and bibliographical work by Baasner (1991) on Kästner, we can concentrate on Kästner's merits as textbook-author and mathematics teacher. In order to embed his *Mathematische Anfangsgründe* within the *Anfangsgründe* tradition, we compare some parts of his textbook with textbooks of other German authors. For this purpose, we choose the four following case studies: classification of the mathematical sciences, negative numbers, the parallel postulate,

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and fortification. The dissertation *Abraham Gotthelf Kästner als Lehrbuchautor. Unter Berücksichtigung weiterer deutschsprachiger mathematischer Lehrbücher für den universitären Unterricht* by Kröger originates from this researches.

3.4 Time

Considering the educational circumstances and the biographical data of the authors of the *Anfangsgründe*, it is easy to determine the period in which the *Anfangsgründe* appeared. The so-called *Anfangsgründe* tradition begins with the release of Wolff's *Anfangsgründe aller mathematischen Wissenschaften* in 1710. Before, mathematical textbooks for the use at universities (for instance, the *Cursus Mathematicus* by Gaspar Schott SJ) were mainly written in Latin. Some textbooks in German existed before the 18th century, but these were not created for teaching the mathematical sciences at universities, but for special addressees, for instance merchants and engineers. Hence, these textbooks were not as extensive as the *Anfangsgründe*, but rather focused on a specific topic, like, for instance, arithmetic. Popular examples are the *Nürnberger Rechenbuch* (1482) by Ulrich Wagner, the *Bamberger Rechenbuch*, and the arithmetic book by Adam Ries (cf. Pahl 1913, p. 80).

Wolff's *Anfangsgründe* dominated until the middle of the 18th century. Then, new textbooks appeared. In the first half of the 18th century, the knowledge about mathematical topics increased, so that the contents of the textbooks had to be adapted. Wolff did not integrate new knowledge (cf. Sommerhoff-Benner 2002, p. 39). So, Wolff's textbook was not up to date in the middle of the 18th century. This fact was also remarked by the contemporaries (cf. for instance Clemm 1777b, preface to the first edition, without page reference).

Already at the end of the 18th century, we observe a change in the *Anfangsgründe* tradition. Extensive textbooks covering all mathematical disciplines are replaced by textbooks dealing with one specific mathematical topic, as for example the *Anfangsgründe der Arithmetik, Geometrie und Trigonometrie* (²1792) and the *Anfangsgründe der Astronomie, nebst der mathematischen Geographie, Schiffahrtskunde, Chronologie und Gnomonik* (1793) both by Georg Simon Klügel. A possible explanation for this development might be the changes within the hierarchy of the mathematical sciences. Some applied mathematical disciplines were outsourced in the 19th century (cf. Stichweh 1984). To these belong the physical sciences which were referred to as applied mathematics in the 18th century. Therefore, new textbooks on the physical sciences were published, for instance the *Anfangsgründe der Naturlehre* (1772, ²1777, ³1784, ⁴1787, ⁵1791, ⁶1794) by Johann Christian Polykarp Erxleben (1744-1777) and

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the *Anfangsgründe der Naturlehre* (1801, ^{2}1805, ^{3}1812) by Johann Tobias Mayer (1752-1830). Also other disciplines were extensively presented in separate textbooks, like fortification. In the *Anfangsgründe* we studied, fortification is usually just treated briefly. In the second half of the 18th century, Karl August von Struensee (1735-1804) published the three-volume *Anfangsgründe der Kriegsbaukunst* (1771-1774, ^{2}1786-1789) for his own lectures at the knight academy (Ritterakademie) in Liegnitz.

Another aspect regarding the shift in the tradition of the *Anfangsgründe* concerns the educational system. New school forms were created to prepare for university and the *Abitur* was instituted as university entrance qualification, for instance in Prussia in 1788 (cf. Kühn 1987, p. 43). Therefore, the arts faculty lost its propaedeutic role. Basics were no longer taught at universities, but at schools (cf. Klein 1968, p. 81). Moreover, there was a major educational reform in Prussia in 1810. The reformers discussed new curricula and suitable textbooks. For the new needs, the *Anfangsgründe* were too comprehensive.

On the basis of these development, we can infer that the *Anfangsgründe* dominated in the 18th century. These textbooks were widely used and met the requirements of the time, especially those of the Enlightenment. The popularity of the *Anfangsgründe* can be explained with the fact that they were versatile, both for various levels and different kinds of schools. The changes within the educational system at the end of the 18th and the beginning of the 19th century can account for the replacement of the *Anfangsgründe* by new, more suitable textbooks.

3.5 Intention and Addressed Public

At the beginning of the *Anfangsgründe,* we usually find the mathematical basics. Since a lot of students had no mathematical training before attending university, the authors of the textbooks began with basics, for instance with the explanation of the numbers and the four arithmetic operations and, then, went on to higher mathematics. Due to this, the *Anfangsgründe* could be used by students at any level, and also by teachers as lecture notes. The latter aspect is also interesting. One must consider that a lot of teachers of the arts faculty, who had to teach the mathematical sciences, were no mathematicians. A lot of professors occupied positions within the arts faculty until they found a better paid position in one of the higher faculties (cf. Turner 1975, p. 499). In addition to lecturers at universities, also the many private teachers were potential addressees of the *Anfangsgründe* (cf. Kühn 1987, p. 40).

The main purpose of the authors of the *Anfangsgründe* was the dissemination of mathematical knowledge: the mathematical sciences should be presented clearly, explicitly,

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and briefly (cf. Clemm 1777b, preface to the first edition p. 5^{r}). Kästner had the aim to present the contents in his textbook in a way that would not deter beginner of mathematics (cf. Kästner 1800, preface to the first edition, p. *4^{v}). He wanted to lay the foundation for further mathematical studies and to go further than Wolff did by aiming at "leading apprentices to the point that they can increase their knowledge by own diligence, and that they can apply their knowledge".[7]

Beyond the dissemination of mathematical knowledge, the authors also pursued other ideals which are directly linked to the educational circumstances during the 18th century. In the prefaces of the textbooks, the authors usually remark three aspects which were regarded as important concerning education since ancient times. Already at the time of Aristotle, higher education should accomplish three purposes: firstly, knowledge and insight, secondly, moral and virtue, thirdly, social needs, utility, and vocational training (cf. Rüegg 1996, p. 53). The first was the main purpose of the authors of the *Anfangsgründe*. The second and the third aims are linked to establishing mathematics as an independent discipline by demonstrating its value for the education in general and in everyday life. Besides some remarks in the prefaces of the *Anfangsgründe*, we can find some writings with meaningful titles. To these belongs Kästner's inaugural speech "*De eo Quod Studium Matheseos Facit ad Virtutem"* from 1756. Kästner illustrated how diligence, which is needed for learning mathematics, can enhance the virtue. Studying mathematics could sharpen your mind (cf. Kästner in Ebel 1978, pp. 55-63), which was an important aspect for Wolff, too (cf. Wolff 1737, preface, p.)(3^{v}). In order to reach this aim, he applied the mathematical method ("*Mathematische Lehrart*"[8]) in the presentation of the contents in his *Anfangsgründe* and his *Auszug aus den Anfangsgründen*.

Kästner also wrote articles about the utility of mathematics for other sciences and everyday life, for instance *Ueber den Gebrauch des mathematischen Geistes außer der Mathematik* and *Ueber den Werth der Mathematik, wenn man sie als einen Zeitvertreib betrachtet.* These speeches and articles indicate the position of the mathematical sciences. It appears that mathematics was still regarded as an ancillary discipline, so that Kästner – and also other authors – saw the need to emphasize its wide usefulness.

Segner considered both the usefulness of the mathematical sciences and the sharpening of the mind in his *Anfangsgründe*: "Everything that is useful, clear, and lofty within the philosophical sciences, we owe largely to mathematics. Through mathematics, the-

7 Translated by Desirée Kröger. Original quote in Kästner, 1800, preface to the first edition, p. *3^{V}: "[…] *Lehrlinge so weit […] führen, daß sie ihre Kenntniß durch eigenen Fleiß erweitern und anwenden können"*.

8 This is the order in which mathematical contents are presented. For a comprehensive explanation of the so-called mathematical method cf. Wolff 1775, vol. 1, pp. 5-32, and Kästner 1800, pp. 1-23.

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se arts were invented or attained perfection, which make our life more convenient. Mathematics, among all other scinces, shows us the way to trueness and certainty most clearly […]”.[9]

The need of sharpening the mind was seen by Clemm, too. He wrote in his *Erste Gründe* that the mathematical sciences are useful for the training of the mind (cf. Clemm 1777a, preface to the first edition, p. 5). For Clemm, mathematics is a discipline which helps to understand other sciences better. But it is also important for the thoughts in general, because it collects one’s thoughts, increases attention, and makes possible to find new inventions (cf. Clemm 1777a, p. 3).

It is obvious that the authors of the *Anfangsgründe* did not only concentrate on imparting knowledge, but also focused on the utility of the mathematical sciences for other sciences, education, and everyday life. It seems to have been a common intention during the 18th century, because all authors we studied – except for Klügel – considered these aspects. Initially in the 18th century, the mathematical sciences was part of the arts faculty and not popular among students. In order to establish the mathematical sciences as an academic discipline and to attract students, the authors provided arguments in favor of learning mathematics. Such arguments we miss in Klügel’s textbook, so that we can assume that mathematics was recognized as an independent discipline by that time. Another evidence might be the establishment of mathematical journals like the *Leipziger Magazin für reine und angewandte Math*ematik (1786-1789) and the *Archiv der reinen und angewandten Math*ematik (1795-1800), which were published by Carl Friedrich Hindenburg (1741-1808), one of Kästner’s students.

The *Anfangsgründe* considered in this work were primarily composed by university professors. So, the main audiences were students. From the meaning of the term *Anfangsgründe* we can deduce that these textbooks were also intended for those students who began their mathematical studies without any previous knowledge. But also students with mathematical knowledge could use the *Anfangsgründe*, for instance in order to learn higher mathematics or the applied mathematical sciences which are also represented in the textbooks.

Wolff wrote his *Anfangsgründe* not only for the use at universities, but also for other schools (cf. Wolff 1775, vol. 1, title page). He did not confine to a certain readership,

9 Translated by Desirée Kröger. Original quote in Segner 1764, preface, without page reference: “*Was in der Philosophie brauchbar, gründlich, erhaben ist, haben wir gröstentheils der Mathematick zu dancken. Durch sie sind die Künste, welche uns die Bequemlichkeiten des Lebens verschaffen erfunden, oder zur Vollkommenheit gebracht worden. Sie zeigt uns unter allen Wissenschaften am deutlichsten den Weg zur Warheit und Gewisheit* […]*”.*

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but wanted "to meet the requirements of all learners according to their different intentions".[10]

Besides the students at universities, also others could use the *Anfangsgründe*, for example professors or teachers as lecture notes, and learners for autodidactic studies. For the latter audience, the German language is an important aspect. As seen above, German became a scientific language in the 18th century. By the use of the German language in their textbooks, the authors could not only counteract the lack of German textbooks, but also reach a broader audience, including those who did not know Latin and were outside of the academic milieu. The authors mention the aim to reach a broader audience in the prefaces of their textbooks as well (cf. Wolff 1775, vol. 1, preface, p. b2v).

Clemm addressed autodidacts explicitly. He wrote his *Erste Gründe* mainly for those "who want to read mathematics in German on their own without oral teaching, and familiarize themselves with it".[11] Also Segner addressed autodidacts, as it is stated in the subtitle "*Zum Gebrauche derjenigen, welche sich in diesen Wissenschaften durch eigenen Fleiß üben wollen*"[12] of his *Deutliche und vollständige Vorlesungen*. This textbook was supposed to be structured in such a way that even children could understand the contents (cf. Segner 1767, preface to the first edition, no page). Segner pursued another aim: "The purpose of the present book was to assist those who want to learn the basics of mathematics through their own diligence, or under the guidance of a teacher, who himself has not fully mastered the same: but even to make the revision of the oral presentation easier for others, and to complement the presentation, if necessary".[13] The explanations in Segner's textbooks should therefore add knowledge to the teacher's lessons. Considering the fact that the positions within the arts faculty at German universities during the 18th century were often temporary, it seems natural that these positions were not always held by experts in the mathematical sciences. They could use the *Anfangsgründe* as lecture notes in order to guarantee a proper teaching.

Karsten wrote his *Lehrbegrif* mainly for beginners of mathematics at universities who should use the textbooks as supplement to the lectures (cf. Karsten 1767, preface,

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10 Translated by Desirée Kröger. Original quote in Wolff 1775, vol. 1, preface to the first edition, p. br: "*allen Lernenden nach ihren gantz verschiedenen Absichten eine Gnüge thun*".

11 Translated by Desirée Kröger. Original quote in Clemm 1777a, preface to the first edition, p.)(2r: "*welche ohne mündlichen Unterricht für sich allein die Grössenlehre in deutscher Sprache lesen, und sich bekannt machen wollen*".

12 "For those who want to practice these sciences on their own". Translated by Desirée Kröger.

13 Translated by Desirée Kröger. Original quote in Segner 1767, preface to the first edition, without page reference: "*Der Zweck bey der Ausfertigung des gegenwärtigen Buches war, denenjenigen, welche sich die Anfangsgründe der Mathematik durch eigenen Fleiß, oder unter der Anführung eines Lehrmeisters, der selbst nicht allzuweit in denselben gekommen ist, bekant machen wollen, dazu beförderlich zu seyn: andern aber die Wiederhohlung des mündlichen Vortrages zu erleichtern, und denselben, wo es nöthig ist, zu ergänzen*".

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p. **r). His textbook was also addressed to those who wanted to acquire mathematical knowledge beyond the basics (cf. Karsten 1767, preface, p. **2r).

Our remarks show that the addressees of the *Anfangsgründe* were beginners in the broadest sense who learned the mathematical sciences at university, at school, or through autodidactic studies. Because of the missing compulsory education in 18th century Germany, a lot of students came in contact with mathematics for the first time at university. Although mathematics was taught in some schools, the focus there was laid on arithmetic skills. Not only students and pupils, but also professors and teachers could use the *Anfangsgründe* as lecture notes. Also home tutors can be regarded as potential readers of the *Anfangsgründe*, because private education was common during this time (cf. Kühn 1987, p. 40). It is noticeable that the authors of the *Anfangsgründe* did not focused on one specific group of readers. The textbooks could be used by different people for various purposes. The variety of the addressees and the versatility can be seen as characteristic of the *Anfangsgründe*. These features might also explain why these textbooks were widespread and often used. This can be regarded as an advance towards the standardization of the mathematical sciences teaching in 18th century Germany, where a uniform and differentiated educational system did not exist. At the end of the 18th and the beginning of the 19th century, first differentiations can be noticed. In the titles of the corresponding textbooks, the addressed readership is often mentioned more precisely. An example is *Anfangsgründe der Differenzial- und Integral-Rechnung zum Gebrauch des Ingenieurs und Artilleristen* (1784) by Christian Karl August Ludwig von Massenbach (1758-1827). This textbook was mainly written for engineers. We also find textbooks which were written for a particular school grade, for instance *Anfangs-Gruende der Rechen-Kunst in Bruechen, welche der Pfoertnischen Jugend in der letzten Classe vorgetragen* (1745) by Johann Georg Gotthelf Hübsch.

3.6 Contents and Structure of the Textbooks

One can find mathematical *Anfangsgründe* on almost all mathematical topics. We focused on those textbooks which were both popular and contain several mathematical disciplines. Not only pure mathematics was included, but also applied mathematics in the form of mechanical, optical, astronomical, and architectural sciences. Some of the textbooks comprise various volumes in which all the mathematical disciplines that belonged to the mathematical sciences in the 18th century were presented. Thus, the *Anfangsgründe* can be called encyclopedic.

The title pages of the textbooks contain some information for the reader (cf. for instance Illustration 8). Initially, the title and the treated topics are stated. Up next, the

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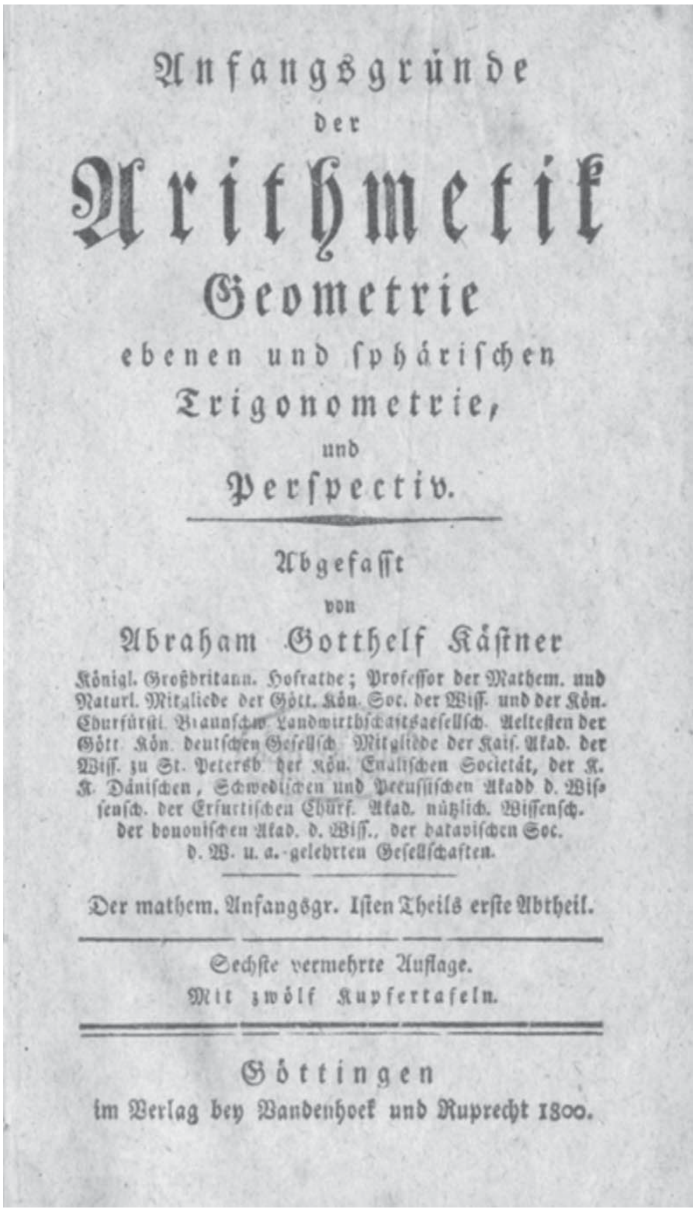
author is mentioned. In the case of Kästner's first volume of his *Anfangsgründe*, Kästner's memberships in academies and societies are listed. This information should indicate the scientific value of the textbook (cf. Baasner 1991, p. 10). Furthermore, we can find information on the volume, the edition, the quantity of copper plates, the place of publication, the publishing house, and the year of publication.


Anfangsgründe
der
Arithmetik
Geometrie
ebenen und sphärischen
Trigonometrie,
und
Perspectiv.

Abgefasst
von
Abraham Gotthelf Kästner
Königl. Großbritann. Hofrathe; Professor der Mathem. und Naturl. Mitgliede der Gött. Kön. Soc. der Wiss. und der Kön. Churfürstl. Braunschw. Landwirthschaftsgesellsch. Aeltesten der Gött. Kön. deutschen Gesellsch. Mitgliede der Kais. Akad. der Wiss. zu St. Petersb. der Kön. Englischen Societät, der K. K. Dänischen, Schwedischen und Preussischen Akadd. d. Wissensch. der Erfurtischen Churf. Akad. nützlich. Wissensch. der bononischen Akad. d. Wiss., der batavischen Soc. d. W. u. a. gelehrten Gesellschaften.

Der mathem. Anfangsgr. Isten Theils erste Abtheil.

Sechste vermehrte Auflage.
Mit zwölf Kupfertafeln.

Göttingen
im Verlag bey Vandenhoek und Ruprecht 1800.

Illustration 8: Title page of the first volume of Kästner's Mathematische Anfangsgründe

Normally, one finds a more or less detailed table of content at the beginning of each textbook: in some textbooks, one can also find dedications. Afterwards, the preface usually follows. In it, the author normally stated his intention and named the intended audience. Kästner, for instance, formulated a new preface for each new edition of his

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Anfangsgründe in which he described the changes in contrast to the previous edition. These changes include, but are not limited to, the elimination of literal errors, improvements of mathematical proofs, and the integration of new knowledge and recently published literature. These amendments shows that Kästner was interested in bringing his *Anfangsgründe* up to date.

The mathematical *Anfangsgründe* which we analyzed vary in their extent. Kästner's *Mathematische Anfangsgründe* stand out: Table 3 shows the quantity of volumes of Kästner's *Anfangsgründe*. At the beginning, the textbook consisted of six volumes. In the course of years, new volumes were added. The integration of these new volumes was easily achievable, because Kästner separated his textbook into different parts and subdivisions from the beginning. By means of this separation, a certain hierarchy of mathematical disciplines can be perceived. In the first part of his textbook, Kästner dealt with elementary pure mathematics. In the second part, applied mathematical sciences are treated. The third part of the textbook deals with higher elementary mathematics in the form of algebra and analysis. On the basis of this knowledge, higher applied mathematics can be learned, which is handled in the fourth part of Kästner's *Anfangsgründe*.

The comprehensive *Anfangsgründe* have in common that arithmetic and geometry (including trigonometry) are presented at the beginning of the textbook. Wolff labeled these two fields as the "two fundamental pillars of the mathematical sciences".[14] These two were regarded as base for all other mathematical sciences.

Not only by means of Kästner's arrangement of his *Anfangsgründe,* but also through some explicit remarks on the mathematical sciences, we can argue that Kästner divided them into classes. In the first volume of the *Anfangsgründe* we can find the chapter "*Vorerinnerungen von der Mathematik überhaupt und ihrer Lehrart".*[15] At the beginning, Kästner distinguished between pure and applied mathematics. He used not only German but also Latin terms: "*mathesis pura vel abstracta*" and "*mathesis applicata*". It is remarkable that he did not use the term "*mathematica mixta*" for the applied mathematical sciences, which was common before the 18th century. We can only assume why Kästner used the term "*applicata*" instead of "*mixta*": maybe the older one was too weak in order to describe the applied mathematical sciences. "*Applicata*" seems more suitable for its character because the doctrines of pure mathematics were applied to problems in nature (cf. Kästner 1800, p. 6).

14 Translated by Desirée Kröger. Original quote in Wolff 1737, preface, p.)(4r*: "*beyden Grund-Säulen der mathematischen Wissenschafften"*.
15 In Kästner 1800, pp. 1-23.

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Part, subdivision	Title	Editions
I, I	*Anfangsgründe der Arithmetik, Geometrie, ebenen und sphärischen Trigonometrie, und Perspectiv*	1758, ^{2}1763, ^{3}1774, ^{4}1786, ^{5}1792, ^{6}1800
I, II	*Fortsetzung der Rechenkunst in Anwendungen auf mancherley Geschäffte*	1786, ^{2}1801 published by Bernhard Thibaut
I, III	*Geometrische Abhandlungen. Erste Sammlung. Anwendungen der Geometrie und ebenen Trigonometrie*	1790
I, IV	*Geometrische Abhandlungen. Zweyte Sammlung. Anwendungen der Geometrie und ebenen Trigonometrie*	1791
II, I	*Anfangsgründe der angewandten Mathematik. Mechanische und optische Wissenschaften*[16]	1759, ^{2}1765, ^{3}1780, ^{4}1792
II, II	*Anfangsgründe der angewandten Mathematik. Astronomie, Geographie, Chronologie und Gnomonik*	1759, ^{2}1765, ^{3}1781, ^{4}1792
III, I	*Anfangsgründe der Analysis endlicher Größen*	1760, ^{2}1767, ^{3}1794
III, II	*Anfangsgründe der Analysis des Unendlichen*	1761, ^{2}1770, ^{3}1799
IV, I	*Anfangsgründe der höhern Mechanik*	1766, ^{2}1793
IV, II	*Anfangsgründe der Hydrodynamik*	1769, ^{2}1797

Table 3: Volumes of Kästner's Mathematische Anfangsgründe

Kästner also distinguished between elementary pure mathematics, that is, arithmetic and geometry, and higher pure mathematics, that is, algebra and calculus. In this context, Kästner explicitly used the term "higher mathematics". Admittedly, he did not use "elementary mathematics", but by using the term "higher" mathematics, we can assume a corresponding classification.

For Kästner, applied mathematics can be separated into four main classes: mechanical, optical, astronomical, and architectural sciences (cf. Kästner 1800, preface, without page reference, and Kästner 1768a, p. 42). Which sciences Kästner subordinated to each class and actually treated in his *Anfangsgründe*, is listed in Table 4.

In comparison to other authors, we can assume that Kästner's hierarchy of the mathematical sciences is representative for the 18th century. He is the only author who gave a concrete classification of the mathematical sciences in his textbook. Nevertheless, we can find all the disciplines that Kästner mentioned in other textbooks with minor

16 The second part of the *Anfangsgründe* was published in two volumes not before the third edition. Previously, the two volumes appeared in one volume with the title *Anfangsgründe der angewandten Mathematik*.

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changes. Special about Kästner's classification is that it was not fixed, but receptive for the integration of new topics or disciplines, respectively. On the one hand, it was possible to integrate new mathematical disciplines, for instance aerometry. Aerometry became a mathematical topic through Wolff's elaboration with the help of arithmetic, geometry, and algebra in 1709 (cf. Wolff 1775, vol. 2, pp. 875 ff.). Wolff's achievement was also honored by Kästner (cf. Kästner 1792b, p. 168). Also Karsten tried to incorporate pneumatics and photometry while he treated them in his *Lehrbegrif*, but we cannot find their treatment in other textbooks. On the other hand, topics could be outsourced from the mathematical sciences. Astrology is an example: it was part of the mathematical sciences before the 18^{th} century, until some scholars, for instance Sturm and Wolff (cf. Sturm 1710, p. 6; cf. Wolff 1716, preface, p. a5^{v}), spoke against its treatment as a mathematical discipline.

	The Mathematical Sciences	
	Pure Mathematics	
Elementary Mathematics		**Higher Pure Mathematics**
Arithmetic		Algebra
Geometry incl. Trigonometry		Calculus
	Applied Mathematics	
Mechanical Sciences: Mechanics, Statics, Hydrostatics, Hydraulics, Aerometry, Hydrodynamics, Artillery		*Astronomical Sciences*: Astronomy, Geography, Gnomonic, Chronology
Optical Sciences: Optics, Catoptrics, Dioptrics		*Architectural Sciences*: Civil Architecture, Fortification

Table 4: Kästner's classification of the mathematical sciences

The various *Anfangsgründe* have different extents. The single volumes run to a few hundred pages. The reader can find a structured presentation. All the mathematical sciences are presented in a way that also people without any previous knowledge could understand. For this purpose, Wolff and Kästner used the "mathematical method" explicitly. Except for Wolff's *Anfangsgründe*, all textbooks contain a more or less detailed table of contents. Each topic is presented independently, but one also finds references to previous explanations. These references are easily implemented because the single chapters consist of numbered sections. Through the references, the coherence of

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the mathematical sciences becomes obvious. In the textbooks by Wolff and Kästner, next to numbered sections, further notations as "*Erklärung*" (definition), "*Lehrsatz*" (theorem) or "*Exempel*" (example) are used. This is a characteristic of the mathematical method that the two authors used in their textbooks.

At the beginning of each chapter, the basic explanations are to be founded. Then, the authors proceed to higher knowledge. There are theorems and their proofs, tasks and solutions, additions, further remarks on the history and on secondary literature, and practical examples. Beyond the explanations, the *Anfangsgründe* contain copper plates in order to illustrate the mathematical contents. Usually, the plates are at the end of the textbook. They can be opened out so that illustration and text lie next to each other.

Related to the structure of the *Anfangsgründe*, one important question was to find out whether it accomplishes the tasks characteristic for textbooks: transfer of knowledge, motivation, joy of learning, exercises, examples (cf. Kuzniar in Wiater 2003, p. 90). In the following remarks, we will also look at these aspects.

In the 18th century, principles of teaching, or didactics, became relevant. This impression is supported by the release of the *Braunschweigische Journal philosophischen, philologischen und pädagogischen Inhalts*, in which contemporaries wrote on educational topics. In this journal, Kästner published an article under the title *Einige Anecdoten aus der Jugendgeschichte des Herrn Hofraths Kästner; ein Auszug aus einem Briefe desselben an den R. Campe* (1788), in which he asked how to motivate a boy to learn although he is not in the mood for it. Kästner seemed to be aware of the existence of different types of learners and sensory channels, and that one should adapt to their needs. As an example, he mentions his father, who did not recognize that Kästner would learn playing music more effectively by reading instead of hearing (cf. Kästner 1788a, p. 41). Thoughts like these are also found in the three-part article by Kästner under the title *Ueber die Art Kindern Geometrie und Arithmetik beizubringen,*[17] which was also published in the *Braunschweigische Journal*. For Kästner, geometrical tasks were motivating because "usually children are in the mood to construct figures, a pleasure, which can be easily transferred to geometric figures".[18] With the help of geometrical constructions it would be possible for pupils to reconstruct the approach and find a proof (cf. Kästner 1788c, p. 263).

17 Part two was published under the title *Fortsetzung des im vorigen Stücke abgebrochenen Aufsatzes: über die Art Kindern Geometrie und Arithmetik beizubringen*. Part three was published under the title *Beschluß des im vorigen Stücke abgebrochenen Aufsatzes: über die Art Kindern Geometrie und Arithmetik beizubringen*. Both articles can be found in the same volume of the *Braunschweigische Journal* as the first part.

18 Translated by Desirée Kröger. Original quote in Kästner 1788c, p. 257: "*gewöhnlich haben Kinder Lust Figuren zu machen, und so läßt sich diese Lust leicht zu geometrischen Figuren leiten"*.

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In order to give an insight into the approach, we will present a mathematical problem from Kästner's *Anfangsgründe* (cf. Illustration 9). The problem is how to construct a triangle on a given straight line. The corresponding figure contains a hint for the solution process. Not only the line segment *AB* and the equilateral triangle *ABC* are given, but also the extension of the line segment and two circular segments. Thus, the reader describes the solution process and even the solution of the problem. In the corresponding solution, Kästner describes the procedure: construct circles around the points *A* and *B* with the radius of *AB*. The circles intersect in a point *C*. The point *C* should be connected with the points *A* and *B* in order to get the triangle.

1. Satz. Aufgabe.

Ueber einer gegebenen Linie AB ein gleichseitiges Dreyeck zu beschreiben 29. Fig.

Aufl. Man beschreibe aus den Mittelpuncten A, B, mit den Halbmessern AB, BA, Kreise, die einander in C schneiden; (2. Foder.) und ziehe nach C von A, B, gerade Linien; (1. Foder.) so ist ACB das verlangte Dreyeck.

Bew. Wenn es einen Durchschnitt C giebt, so ist AC = AB (13. Erkl.) = BC;

Daß es aber einen Durchschnitt giebt, erhellt so: Man verlängere AB, bis BD = AB (1. Foder.) so geht der Kreis um B, durch D (13. Erkl.) einen Punct auserhalb des Kreises um A, (weil AD = 2AB) und auch durch A, einen Punct innerhalb desselben; folglich schneidet er jenen Kreis (7. Grundſ.).

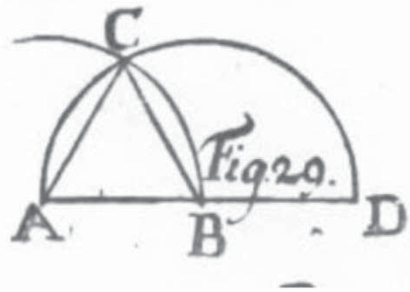

Illustration 9: Kästner 1800, p. 190 and fig. 29

The illustrations and construction problems in Kästner's *Anfangsgründe* have different functions. They are an important feature within the learning process. The images should illustrate the theoretical explanations: there is a close relation between text and image. Beyond that, geometric constructions executed by the pupils have a motivating function. Moreover, they provide an opportunity for pupils to find a theorem or a proof independently through their own activity. This idea corresponds to one of the aims of the Enlightenment, namely learning by insights instead of by rote memorizing. Our observations also applies to the *Anfangsgründe* in general. In all textbooks, we can find a lot of practical examples. So, the readers can connect theory and experiences from everyday life in order to facilitate the mathematical sciences learning. The various plates and illustrations are helpful for the visualization of the theoretical remarks.

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Thus, we can say that the *Anfangsgründe* accomplish the tasks which are important for textbooks: motivation, joy of learning, exercises, and examples.

3.7 Further Characteristics

In the following, we will present some further characteristics of the *Anfangsgründe* while laying the focus on Kästner's textbook. These characteristics are, firstly, the interactive relation between textbook and teaching, secondly, the connection between textbooks and research.

Due to the fact that the *Anfangsgründe* were written as lecture notes, it is possible that the teaching and the textbooks were geared to each other, namely that teaching experience affected the textbooks. Kästner made some interesting remarks on this topic in the prefaces of his *Anfangsgründe*. On the one hand, Kästner presented some contents at first only in his lectures, but then decided that it would be useful for the students to find them also in the textbook (cf. Kästner 1800, preface to the fourth edition, without page reference). On the other hand, he treated some contents in a more detailed way in his *Anfangsgründe* in order to save time during the lectures (cf. Kästner 1792a, preface to the third edition, p. v). The students had the possibility to learn on their own, to prepare themselves, or to revise afterwards.

Kästner dovetailed teaching and textbook, and wanted to optimize this relation. Because of his teaching experience, Kästner was engaged in questions concerning the students' understanding of mathematical problems (cf. Kästner 1788b, p. 390). Beyond his own experiences, Kästner got various feedback from colleagues and friends, so that he could improve his textbook, for instance by the elimination of literal mistakes, improving proofs or more detailed explanations (cf. Kästner 1800, preface to the third edition, p. **3^{v}). As mentioned above, Kästner used images in order to illustrate the contents. Moreover, during his lectures on applied mathematics, he used further visual tools like instruments or models (cf. Kästner 1768b, pp. 45 ff.). Due to the fact that Kästner supplemented his lectures with these tools, we can assume that his lectures and his *Anfangsgründe* were geared to each other. He improved his textbook based on his own teaching experience and on the experiences of colleagues and friends.

Kästner was not only professor of mathematics and physics at the University of Göttingen, but also interested in didactics. In the last third of the 18th century, the general interest in pedagogy was high. This is revealed by the increasing publications of journals on pedagogy. Worth mentioning are, for instance, the journals *Allgemeine Bibliothek für das Schul- und Erziehungswesen in Deutschland* (1773-1784/86), *Magazin für öffentliche Schulen und Schullehrer* (1790-1791), and *Magazin für Schulen*

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und die Erziehung überhaupt (1766-1771/72). The magazine *Braunschweigisches Journal philosophischen, philologischen und pädagogischen Inhalts* (1788-1791) is of peculiar interest. There, Kästner published two articles on didactic elements in mathematics education: *Einige Anecdoten aus der Jugendgeschichte des Herrn Hofraths Kästner; ein Auszug aus einem Briefe desselben an den R. Campe* and *Ueber die Art Kindern Geometrie und Arithmetik beizubringen* (in three parts). Kästner wrote about issues which he regarded important for mathematics education. Thereby, he reflected his own childhood and education. He argued against rote memorizing and spoke for learning through insights (cf. Kästner 1788c, p. 262). There was a shift in the 18th century: dissociation from the dogmatic teaching method to which rote memorizing was central (cf. Sommerhoff-Benner 2002, p. 318). This development fits into the Age of Enlightenment. The enlighteners demanded: "*Sapere aude! Habe Muth dich deines eigenen Verstandes zu bedienen!*".[19] This corresponds to the new teaching prospective of training the mind.

Kästner emphasized the importance of motivation for the learning process. Thereby, the teacher should present the contents clearly and vividly. Kästner did not only intend to motivate his students with some images and constructions, but also with the help of some demonstrations during his lectures. Especially in his lectures on applied mathematics, Kästner used models and instruments as illustrations. Kästner owned some instruments himself and could also use those of the University of Göttingen (cf. Pütter 1765, p. 300).

In the 18th century, teaching and research were independent of each other. While universities were responsible for teaching, research was conducted at scientific societies (cf. Grau 1988, p. 16). The authors of the *Anfangsgründe* that we studied were no researchers. The other way around, popular mathematicians as Leonhard Euler (1707-1783) did not have a teaching position at a university, but worked at an academy. At least since the Prussian educational reform of 1810, teaching and research were combined at universities. Nevertheless, there had been an endeavor at the universities in Halle and Göttingen to combine teaching and research before (cf. Schindling in Hammerstein 1995, p. 17). That implies that the demarcation between teaching and research already blurred in the 18th century. We studied the *Anfangsgründe* in order to find out if we can detect any hints for this hypothesis.

Our assumption derives by the fact that Kästner and other authors of the *Anfangsgründe* dedicated some detailed paragraphs to the parallel postulate, which at

19 Kant 1923, p. 35. "*Sapere aude!* Have the courage to make use of your own reason". Translated by Desirée Kröger.

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the time was a research issue. Scholars still tried to provide a proof of it, until the development of the non-Euclidean geometries in the 19th century by Carl Friedrich Gauß (1777-1855), Nikolai Lobatschewski (1792-1856), and Johann Bolyai (1802-1860) (cf. Stäckel/Engel 1895, preface, p. iii). The most comprehensive remarks on the parallel postulate can be found in Kästner's *Anfangsgründe*. Already in the preface, he informed the reader about the problems connected to the postulate and his own attempts to proof it. He wrote that he had worked on the parallel postulate for many years and that he had collected a lot of sources in order to find the solution for this problem, but he had failed. Kästner emphasized that he was not able to give a proof for the postulate in his textbook, but that he would present a reasonable treatment of the postulate with the help of some remarks (cf. Kästner 1800, preface to the first edition, pp. *5^v ff.). At this point, he referred to Klügel's dissertation *Conatuum Praecipuorum Theoriam Parallelarum Demonstrandi Recensio* (1763) which had arisen from Kästner's suggestion (cf. Kästner 1800, preface to the first edition, p. *5^v, footnote (*)). In this work, Klügel gave a historical overview of the postulate and discusses 28 attempts[20] of proving it. The fact that Kästner and other authors of the *Anfangsgründe* mentioned the parallel postulate is an indicator that they wanted to sensitize their public to mathematical research problems. With the help of his *Anfangsgründe*, Kästner did not only want to teach the mathematical sciences, but also wanted the reader to be able to discover new mathematical results (cf. Kästner 1800, p. 6). This might explain why he also mentioned those mathematical topics and problems which were beyond the scope of normal elementary subjects. By this means, the *Anfangsgründe* get a scientific feature.

Another observation related to the mixity in the 18th century between teaching and research is that the authors of the *Anfangsgründe* did not only write textbooks, but also research material. The fact that the mentioned authors wrote on mathematical research problems beyond their teaching-positions shows that they considered themselves not only as teachers, but also as researchers. The parallel postulate serve again as an example. As mentioned above, Kästner worked on a proof for this postulate many years long, and Klügel published his dissertation on this topic. Beyond the critical remarks in his textbooks, Karsten wrote on the parallel postulate in his *Beyträge zur Aufnahme der Theoretischen Mathematik* (4 volumes, 1758-1761), namely in the chapter "*Von dem 13ten Grundsaz* [sic] *im 1sten Buch der Elementorum Euclidis*". The *Beyträge* contain topics on pure and applied mathematics. They were composed for students and people with similar mathematical prior knowledge, and students who wanted to learn more about the mathematical sciences (cf. Karsten 1758, preface, p. 4). Also Karsten

20 In the secondary literature, the number of thirty proofs is often erroneously mentioned.

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dealt with the theory of parallels: he dedicated his inaugural speech "*Versuch einer völlig berichtigten Theorie von den Parallellinien*" (1778) for the professorship at the University of Halle to it. There, he mentioned that he had tried to prove the parallel postulate, but that he was not satisfied with the results (cf. Karsten 1778, p. 10).

There is another hint to the fact that the treated authors were interested in the progress of the mathematical sciences. Indeed, beyond their teaching positions, they were usually also members of many scientific academies.[21] It is possible that the authors wanted to sensitize to mathematical problems by integrating advanced mathematical issues, as for instance the explanation of the parallel postulate. One overall feature of the *Anfangsgründe* is to provide a basis for further studies on the mathematical sciences and many references to further literature can be found in these textbooks, so that students could work on them independently. Kästner did not only refer to other works, but also used scientific monographs (for instance, by Euler) as refrence for algebraic topics (cf. Kästner 1794, preface, pp. vii f.). For his *Anfangsgründe der höhern Mechanik*, Kästner used the monographs by Euler, Bernoulli, and d'Alembert (cf. Kästner 1793, preface to the first edition, pp. vii ff.) and, for his *Anfangsgründe der Hydrodynamik*, he took the works by Euler and Bernoulli as a basis. In doing so, he did not adopt the contents as they stood, but gave some own remarks (cf. Kästner 1797, preface, p. *3^v f.). By this procedure, Kästner introduced mathematical research contents in his *Anfangsgründe*. We can assume that Kästner adjusted the contents from the scientific monographs so that beginners of the mathematical sciences – his main audience – could understand them. This procedure is called "didactic transposition" (cf. Chevallard 1991, pp. 39 ff.).

In the course of our research on the *Anfangsgründe*, we could conclude how long a teacher should employ to deal with the contents. These textbooks were mainly created to assist teaching the mathematical sciences at universities. So, we can assume that one volume had been created for the duration of one semester. Not all authors commented on this issue. Karsten wrote in his *Lehrbegrif* that he conceived the first two volumes of this textbook, which contain elementary mathematics, for the duration of one semester each (cf. Karsten 1768, preface, p. *5^v.). Also the three volumes of his *Anfangsgründe* had been written for one semester each (cf. Karsten 1780, vol. 1, preface, p. X f.). He stated in the preface to his *Anfangsgründe der angewandten Mathematik* that he wanted the contents to be learned within one semester (cf. Kästner 1792b, preface to the first edition, p. *2^v). Due to the fact that the extent of the single

21 For Kästner, the title page of the first volume of his *Anfangsgründe* contains a list with his memberships in scientific societies and academies.

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books, or volumes respectively, of the studied *Anfangsgründe* is comparable with Karsten's textbooks, namely around 500 pages per volume, we can assume that one volume of these textbooks were conceived for the duration of one semester, too.

3.8 Usage and Dissemination

Due to the fact that the *Anfangsgründe* were written in German, it is obvious that these textbooks were mainly used in Germany. They were also translated into other languages. Wolff's *Anfangsgründe* was translated into Dutch, French, Polish, Russian, and Swedish (cf. Sommerhoff-Benner 2002, p. 40) and, in 1794, a free Russian translation of the first volume of Kästner's *Anfangsgründe* appeared (cf. *Göttingische Anzeigen von gelehrten Sachen* 1796, 177. St., p. 1766). This is probably due to a lack of suitable mathematical textbooks in some of these countries, but we must also admit that the *Anfangsgründe* were so popular that they were known abroad and translated into the respective national languages.

Concerning the dissemination of the *Anfangsgründe*, we consider firstly the numbers of editions. Kästner's first volume of the *Mathematische Anfangsgründe* was published in six editions. Wolff's *Anfangsgründe* was published until 1800 – a long time after his death. Clemm was surprised about the fact that his *Mathematisches Lehrbuch* had to be reprinted only three years after the first edition in 1764 (cf. Clemm 1777b, preface to the second edition, p.)( )(^{r}). The numbers of the editions are in fact an indicator for the demand for those textbooks in the considered period. Kästner wrote in his *Commentarius* that there were only few people who wanted to learn the mathematical sciences (cf. Kästner 1768b, p. 40). It seems therefore that there was a shift in the course of the 18^{th} century; otherwise, we could not explain the popularity of the *Anfangsgründe*. Kästner wrote that not only his own, but also the textbooks by Karsten and Segner on pure elementary mathematics appealed to the German population (cf. Kästner 1800, preface to the second edition, p. **^{r}).

Lecture catalogs can give more evidence for the usage of these textbooks.The analysis of the lecture catalogs at the University of Leipzig shows that the textbooks by Wolff, Segner, Kästner, and Karsten were there widely used (cf. Kühn 1987, p. 79). Our analysis of the lectures catalogs of the University of Göttingen until 1800, which one can find in the university and academy journal *Göttingische Anzeigen von gelehrten Sachen*, shows that Kästner's textbooks were most frequently used. Indeed, they were leading at this university in the second half of the 18^{th} century (cf. Klein 1968, p. 81). Beyond Kästner's *Anfangsgründe*, Wolff's textbooks were still widely used at the University of Göttingen.

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Kästner's textbooks were regarded as common (cf. Müller 1904, p. 58). We have also studied some lectures catalogs from other universities and inferred that Kästner's *Anfangsgründe* were also used at the universities of Braunschweig, Freiburg, Gießen, Ingolstadt/Landshut, and Kiel (cf. Kröger, forthcoming, chapter 3.4). Concerning the dissemination of Kästner's *Anfangsgründe*, Cantor stated that every German mathematician knew his textbook (cf. Cantor 1965, vol. 4, p. 1096).

Due to the lack of source material, it is not easy to find out if the *Anfangsgründe* were as a matter of fact used by the supposed audience. Outside of the academic milieu, it was difficult to trace who used the *Anfangsgründe*. For this purpose, we studied both the secondary literature and some biographies of mathematicians of the 18th century. Beyond the main audience, namely the students, we discovered that the *Anfangsgründe* were used in other school forms, and also for autodidactic studies. Kästner, for instance, never attended a public school, but was privately educated by his father and his uncle. He learned mathematical calculations independently with the help of Wolff's *Auszug* (cf. Kästner 1768a, pp. 46 ff.). During his days at the Gymnasium, Gauß bought some textbooks written by Wolff and Kästner (cf. Reich 2005, pp. 79 ff.). Bernard Bolzano (1781-1848) acquired mathematical skills by means of Kästner's *Anfangsgründe* (cf. Bolzano 1810, preface, p. XI). The university and state library of Düsseldorf owns an exemplar of the first volume of Karsten's *Anfangsgründe*. In this, the inscription "*Fr. Benzenberg 1802*" and the stamp "*Benzenberg, Sternwarte der Stadt Düsseldorf 1846*" are found. This shows that Karsten's textbook was part of the inventory of this astronomical observatory until the university and state library of Düsseldorf acquired the inventory, which was destroyed in the Second World War.[22]

Beyond the university milieu, the *Anfangsgründe* could have been used in other kind of schools. Clemm was a teacher in a Gymnasium in Stuttgart and used his textbooks for his lessons. Especially for the use at schools for general education, Wolff published his *Auszug aus den Anfangsgründen* because his *Anfangsgründe* were considered too expensive and too comprehensive (cf. Wolff 1737, preface, p. 4v.). Also Kästner's *Anfangsgründe* were used in a Gymnasium, namely at the grammar school in Darmstadt, where Georg Christoph Lichtenberg (1742-1799), a posterior student and colleague of Kästner, was a pupil (cf. NDB 1985, vol. 14, p. 450).

22 According to the statement of an employee of the library.

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3.9 Further remarks

For two reasons we pay particular attention to Kästner in our research. Firstly, Sommerhoff-Benner (2002) already worked on Wolff, his textbooks, and his merits within the the mathematical sciences teaching. Secondly, we found some statements of Kästner's contemporaries which label Kästner as one of the greatest mathematicians. Johann Andreas Christian Michelsen (1749-1797) called Kästner the teacher of mathematics for the German nation (cf. Müller 1904, p. 58). Other contemporaries regarded Kästner's textbook as a contribution to the completion and extension of mathematical studies (cf. Jördens 1812, p. 55). These remarks let us assume that Kästner set a benchmark within the mathematical education with his *Anfangsgründe*. He did not publish his textbook only for prestige proposes. He rather felt the need to think about the pedagogy of his days, didactic elements, and teaching.

Wolff's *Anfangsgründe* had a monopoly position until the middle of the 18th century (cf. Nobre 2004, p. 13). Then the next generation of mathematic professors published new *Anfangsgründe*. Especially Kästner's *Anfangsgründe* replaced Wolff's textbook (cf. Murhard 1797, p. 71). Our research confirms this thesis, because Kästner's textbook appears as most popular and used. The reasons for the fact that Wolff's *Anfangsgründe* were not common anymore are diverse. Firstly, Wolff published his textbook at the beginning of the 18th century. He could not orientate himself towards other textbooks on the mathematical sciences, because he was the first to publish one in German. So, he also had the task to translate mathematical terms into German. We owe Wolff the germanization of mathematical terms (cf. Nobre 2004, p. 11). Secondly, Wolff did not bring his textbook up to date (cf. Sommerhoff-Benner 2002, p. 39). As a consequence, there was a demand for textbooks which contain also current mathematical knowledge.

Müller claims that Kästner's *Anfangsgründe* were used as a role model for other textbook-authors in the 1770s and 1780s (cf. Müller 1904, p. 58). Since we studied those textbooks which contain various mathematical disciplines, we cannot confirm this thesis completely. Nevertheless, we could find out that Kästner's classification of the mathematical disciplines was used as a role model by Karsten. He explicitly referred to Kästner's remarks and adopted it (cf. Karsten 1767, preface, without page reference). Kästner was the first author who used the term "*angewandte Mathematik*" (applied mathematics). Wolff did not use this term, but spoke of "*angebrachte Mathematik*" (cf. Wolff 1716, col. 866). The authors who published textbooks after Kästner also used the term "*angewandte Mathematik*", for instance Clemm, Karsten, and Klügel. Another top-

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ic for which Kästner can be regarded as a role model is that of the negative numbers. Kästner wrote about them in his *Anfangsgründe* in a particularized way. Thereby, he motivated Immanuel Kant (1724-1804) to write about this topic (cf. Kant 1763, p. 2).

Kästner's *Anfangsgründe* were well received. Karsten denoted Kästner's textbook as "*vortrefflich*"[23] (Karsten 1759, p. 217). He named Kästner one of the greatest mathematicians, especially concerning the topic of calculus (cf. Karsten, 1761, p. 275). Actually, Kästner's *Anfangsgründe* on algebra and calculus (volumes 7 and 8 of the *Anfangsgründe*) had an outstanding role because they were the first German-language textbooks on these topics. They replaced the previously used *Élémens d'Algébre* by Alexis-Claude Clairaut (1713-1765) (cf. Müller 1904, p. 65).

The success of Kästner's *Anfangsgründe* might be related to some features of them. Kästner treated all mathematical disciplines which fell under the umbrella of the mathematical sciences in the 18th century. As well, he arranged the mathematical disciplines within his concrete classification. Thereby, he did not only distinguish pure and applied mathematics, but also elementary (arithmetic and geometry) and higher mathematics (algebra and calculus). The latter distinction we still know nowadays and can be regarded as demarcation between the curriculum of schools of secondary and higher education (grammar schools and universities).

23 Excellent.

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4 Case Studies

To give an insight into the contents of the *Cours* and *Anfangsgründe* textbooks, we present in this chapter four case studies. We selected the case studies from different domains: arithmetic/algebra, geometry, and applied mathematics. The first case study is about negative numbers, the second deals with the Pythagorean theorem, the third expounds the treatment of ballistics, and the fourth concerns fortification. While presenting each case study, we give a short introduction to the topic at the beginning. Then, we take a look at the French and German parts independently at first and, afterwards, in a comparative way.

4.1 Arithmetic: Negative Numbers

4.1.1 Introduction

In the 18^{th} century, a current question was how to interpret negative numbers, either within arithmetic or algebra. In the past, there had already been attempts to find an interpretation. Common interpretations of negative numbers were as possessions and debts, and as opposed directions moving forward and backward (cf. Tropfke 1980, pp. 145 and 148).

For the French part, we consider our usual selection of textbooks. It turns out that the concerned passages are all contained in the algebra parts and that the most meaningful examples of how negative numbers are employed in practice come from quadratic equations.

For the German part, there are three sources that give us an insight into the contemporary discussion and the associated problems with negative numbers. To these belong *Gedanken über den gegenwärtigen Zustand der Mathematik* (1789) by Johann Andreas Christian Michelsen (1749-1797), *Versuch das Studium der Mathematik durch Erläuterung einiger Grundbegriffe und durch zweckmäßigere Methoden zu erleichtern* (1805, published anonymously) by Franz Spaun (1753-1826), and the reaction to Spaun's work, namely *Ueber Newtons, Eulers, Kästners und Konsorten Pfuschereien in der Mathematik* (1807), by Karl Christian von Langsdorf (1757-1834). Spaun criticized, among others, that the plus and the minus signs have a double meaning: they are the signs for the arithmetic operations of addition and subtraction and, also, they denote the algebraic symbols for positive and negative numbers (cf. Spaun 1805, pp. 7 and 18). Spaun also spoke against the usage of the expression "negative" in order to denote negative numbers (cf. Spaun 1805, p. 7). Langsdorf, in contrast, argued that

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this expression is a convention for mathematicians and could be used for negative numbers (cf. Langsdorf 1807, p. 12.).

This case study results from a workshop held at the conference "7th European Summer University on the History and Epistemology in Mathematics Education" in Copenhagen in July 2014. There, we wanted to take a look at some different approaches to negative numbers, and especially at their justifications, in a small selection of German and French textbooks from the 18th century, namely including Bélidor, La Caille, Bézout, Euler, Kästner, and Wolff. After a brief presentation of the German and French circumstances (educational system, institutional conditions, position of the mathematical sciences, textbooks and their authors), we invited the participants to work in teams on the different sources. At the end of the workshop, every team presented their results. The aim was to show the differences among the various approaches to negative numbers at that time, also in comparison with the developments that lead to nowadays approaches. In order to make the study on the sources easier for the participants and to guarantee comparable results, we proposed the following questions for the analysis of the sources:

- Definition: Is there a definition of negative numbers? If yes, where is it located in the textbook? Are there examples to explain the definition? If yes, what are they?
- Terminology: Which expressions are used?
- Are there interpretative models for negative numbers?
- Are there also non-mathematical remarks (philosophical, historical, ...)? Is the difference between plus and minus once as arithmetic operators, once as algebraic signs clear?
- Applications: how are negative numbers used in calculations (subtraction, multiplication in algebra, quadratic equations)?
- Are there parallels or differences to nowadays approaches?

4.1.2 France

All the considered French authors delayed the treatment of negative numbers to the algebra part. Bélidor's approach is in this respect peculiar since he did not deal with elementary arithmetic at all, so that negative numbers are explained right at the beginning of the textbook (Part I). Indeed, he took for granted that his readers were acquainted with calculations with integer and fractional numbers and started, after having stated some basic geometrical definitions (without examples), with calculations

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with "algebraical quantities". This term refers to the fact that, in these quantities, letters are used as signs to point at non-defined numbers. Bélidor maintained that, when an algebraic quantity is preceded by no sign, that is neither by + nor by – , he always supposed that it has the sign + and called it "positive quantity". On the other hand, the quantities that are preceded by the sign – are called "negative" (cf. Bélidor 1725, p. 11). He provided as examples $+ab = ab$ and $-ab$, whereby he denoted the algebraical quantity *ab* through the extremes *a*, *b* of a geometrical segment. Bélidor provided some interpretative models for negative numbers. Firstly, he interpreted them as possessions and debts (cf. Bélidor 1725, p. 14). Later, he stressed that negative quantities are not "less real" than positive ones. Indeed, they are opposite quantities, which means that they have contrary effects in calculations (cf. Bélidor 1725, pp. 18 and 80). Bélidor never clearly stated the difference between plus and minus as arithmetic operators on the one hand, and as algebraic signs on the other. He suggested both viewpoints (cf. respectively Bélidor 1725, pp. 8 and 12-13 and Bélidor 1725, pp. 14 and 18), but he never compared them in a critical way. Negative quantities appear at first while dealing with algebraical subtraction, where a certain $-b$ stands alone. This means that, in this case, the minus sign means that the quantity *b* is negative, and it is not an operation. Many other examples, for instance $(-8abc)(-5bcd)$ and the result of $(a- b)(a-b)$, can be found in the paragraph on algebraical multiplication. In this passage, Bélidor argued that, if the multiplicand has the sign + (respectively –), the multiplication is made by addition (respectively subtraction) of the same algebraical quantity. A classical example concerns the signs rule, namely when one or more negative multiplicands are involved. The most interesting examples, however, are found in the treatment of quadratic equations (cf. Bélidor 1725, pp. 158-166). There, Bélidor gave no general method for solving them, but rather a collection of solved examples. While commenting some of these, he affirmed that a negative root is to be considered a solution of the problem as well as and with the same degree of trustworthiness as a positive one. Again, he stated that negative roots give a solution "in the sense that we intended", meaning that when a negative solution is found, only the interpretation ha to be adapted, for instance in terms of debts. Finally, he remarked that the algebraic values are true and reasoned, even if theysometimes seem not to have a meaning because they exceed the scope of imagination.

In contrast, as all the other authors taken into account, La Caille firstly dealt with arithmetic, then with algebra. Again as all the others, negative numbers only occur in the algebra part. For La Caille, algebra is a kind of arithmetic which is more general, faster, briefer, simpler, and that can be applied in many occasions. Among the preliminary notions, he passed from the definition of "algebraic quantity" quite immediately

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that of “polynomial” (namely, an algebraic quantity that contains more than one term). Here, the sole definition integrating negative numbers is found: La Caille explained that there are two kinds of terms, the positive ones and the negative ones. The latter are always preceded by the sign –, the other by the sign + (cf. La Caille 1741-1750, vol. 1, p. 62). He only gave the example $+p-q-rr+x-y$, where no term preceded by a minus sign stands alone. La Caille interpreted negative numbers as opposite quantities. Indeed, he explained that $-3a$ is a same quantity a taken three times, as for $+3a$, the only difference being that it is taken in the opposite direction. Apart from this and the usual signs rule for multiplication, no other concrete examples can be found. But obviously La Caille is compelled to deal with negative numbers in solving quadratic equations. While giving the general solving method with the quadratic formula, La Caille repeatedly remarked that a solution can be negative (cf. La Caille 1741-1750, vol. 1, pp. 130-135). He even mentioned that square roots of negative numbers can appear. To this purpose, he limited himself to explaining that it is impossible to find a quantity that, being multiplied by itself, gives a negative product, but he added no judgment of value. When the problem that leads to an equation with a negative solution is interpreted in “real” life (for instance, when we search for the number of travelers), a negative solution only points to the fact that also this negative number (for instance, –6) satisfies the equation (cf. La Caille 1741-1750, vol. 1, p. 135). But of course – added La Caille – only the positive solution is the one that we were searching for. Further on, he remarked that, when the result of a calculation gives a negative value for the unknown, this means that one has to take this unknown in the opposite direction compared to the one that they has considered at the beginning (cf. La Caille 1741-1750, vol. 1, p. 291).

In Camus’ textbook negative numbers do not appear. Indeed, it only attains an elementary level. As all the other French authors, Camus considered only positive numbers in the part on arithmetic, and algebra is not included at all in the table of contents.

Bézout’s treatment of negative numbers was highly detailed. Since there are only minor differences between the textbooks for the navy and for the artillery, we take into account the first one. Bézout gave the definition at the beginning of the algebra volume: as usual, the quantities which are preceded by the plus sign are positive, while the ones that are preceded by the minus sign are negative (cf. Bézout 1764-1769, vol. 3, p. 9). No example is given at first, but later on Bézout devoted a whole paragraph to the topic (cf. Bézout 1764-1769, vol. 3, *Réflexions sur les Quantités Positives et les Quantités Négatives*, pp. 78-84). Among the French authors of our selection, he is the only one who explicitly discussed the distinction of + and – as operators and as denotations of properties of quantities. Bézout had already dealt in the usual way with +

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and – as addition and subtraction in the preceding paragraphs of the arithmetic and algebra parts dedicated to these topics. In this paragraph, he focused on the plus and minus signs as “the way of being of quantities, one in regard to the other”. On the one hand, Bézout legitimated the negative quantities with the usual interpretative models, while, on the other hand, he weakened the ontological status of these quantities. The discussion is set observing that the same quantity can be considered from two opposed viewpoints, and the analogies of possessions and debts and opposite directions on a line are presented. Bézout stressed that the negative quantities are as much real as the positive ones, except that they have a opposite “meaning” in calculations. This means that negative quantities have properties opposite to the positive quantities or, equivalently, that they behave in an opposite way. At the same time, Bézout also stated that each negative quantity points to a false assumption in the statement of the problem and, at the same time, at its correction since it is enough to take the quantity in the opposite direction. This paragraph seems to provide a conceptual frame to make students more readily accept negative numbers. When it comes to the practice, for instance with quadratic equations, Bézout had no hesitation in accepting negative solutions (cf. for instance Bézout 1764-1769, vol. 3, pp. 125-128). On the contrary, Bézout states that, when a problem leads in the end to the square root of a negative number, it is impossible since such a root does not exists. Nevertheless, he goes on that these numbers should not be neglected in the solution procedure, because sometimes square roots of negative numbers annihilate two by two.

Finally, Bossut started the discussion in the algebra volume by defining, among others, the plus and minus signs as operations. The first negative quantity (–*b*) appears even before the definition, as it was self-evident. According to Bossut, negative and positive quantities are of the same kind, but they are opposite regarding to their way of being (cf. Bossut 1772-1775, vol. 2, p. 10). He instantiated this definition with two examples from real life which provide as many interpretations. They boil down as usual to possessions and debts and to considering the opposite direction on a line. In the main, Bossut’s textbook reveals several similarities to Bézout’s and, in practice, for instance while dealing with quadratic equations, negative solutions are accepted without reserves. At this point, not only Bossut did not feel the need to define a negative quantity before mentioning it, but even not to extensively justify a negative solution of an equation. Referring to one particular numerical equation with one positive and one negative solution, Bossut briefly said that both numbers solve the equation (cf. Bossut 1772-1775, vol. 2, p. 189). His justification steps out of the intuitive grasping of “real” life towards a more abstract level: the algebraic calculation, in which the two solutions are simply substituted in the equation at issue, is now enough. Nevertheless, the collection

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of examples that follows conforms to the standard justification: when the equation refers to a problem with an interpretation in "real" life, the negative solution (if there is one) is also interpreted as the opposite of the positive one (to gather or to loose water).

4.1.3 Germany

Wolff explained the negative numbers within the algebra chapter in the fourth volume of his *Anfangsgründe*, which deals with solving equations. We cannot find a concrete definition of a negative number or of opposed magnitudes. Wolff did not use the terms "positive", "negative", or "opposed" magnitudes, but described these magnitudes as money, depths, and lack (cf. Wolff 1775, vol. 4, p. 1557). Kästner is the first author who gave a concrete definition of opposed magnitudes within the arithmetic chapter at the beginning of his *Anfangsgründe*. Euler handled these numbers in his textbook on algebra. While there is no concrete definition in Wolff's textbook, Kästner gave a definition of opposed magnitudes: "Opposed magnitudes are called magnitudes from the same kind, which are considered under such conditions that one of them reduces the other one. For instance assets and debts, moving forward and backward. One of these magnitudes, no matter which one, is called positive or affirmative; the opposed magnitude negative or negating".[24] Euler defined negative numbers: "All these numbers, whether positive or negative, have the known appellation of whole numbers, or integers, which consequently are either greater or less than nothing" (Euler 1822, p. 5). Euler extended the definition of negative numbers by attributing them to a concrete number range, namely the integers. In Euler's definition, another interesting aspect is found. This concerns the expression "less than nothing". In the 18th century, the interpretation of negative numbers was still discussed. From a philosophical point of view, it is difficult to label negative numbers as "less than nothing", because they are real entities, for instance debts. Therefore, Kästner saw the need to explain the expression "less than nothing" in his textbook. He motivates the distinction between an "absolute nothing" and a "relative nothing". Concerning the negative numbers, the relative nothing has to be chosen because a negative number or magnitude can only exist because of its opposed (positive) magnitude. It would be wrong to call a number negative in an absolute meaning (cf. Kästner 1800, pp. 72-74). Euler equated "nothing" with the number "zero" and showed, with the help of a number line, the positive and negative numbers (cf. Euler 1822, p. 5).

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24 Translated by Desirée Kröger. Original quote in Kästner 1800, p. 71: "*Entgegengesetzte Grössen heissen Grössen von einer Art, die unter solchen Bedingungen betrachtet werden, daß die eine die andere vermindert. Z. E. Vermögen und Schulden, Vorwärtsgehen und Rückwärtsgehen. Eine von diesen Grössen, welche man will, heisst man positiv oder bejahend, die ihr entgegengesetzte negativ oder verneinend*".

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Contemporaries claimed that Kästner defined the concept of negative numbers quite well. Kästner devoted a whole paragraph (§ 95) to the nature of negative numbers. With his remarks on the nature of negative numbers and the notion of "less than nothing", he even impressed the philosopher Immanuel Kant (1724-1804), who wrote about negative numbers in his work *Versuch den Begriff der negativen Größen in die Weltweisheit einzuführen* (1763).

Although Kästner was very ambitious to clarify the nature of negative numbers, he did not explain the negative numbers completely in his *Anfangsgründe*. But, in this article *Ueber eine scheinbare Schwierigkeit vom kleinern und grössern, bey Quotienten*, Kästner explained in detail that each negative number is less than each positive number (cf. Kästner 1787, p. 71). We cannot find this relation in his textbooks. This indicates that the contents were not intended for textbooks. We can only assume why Kästner did not mention them in his textbook. Maybe he did not want to overexert beginners of the mathematical sciences, which was one of his general intentions (cf. Kästner 1800, preface to the first edition, p. $*4^{v}$). It is also possible that this topic was not a unit for elementary pure mathematics.

Wolff explained negative magnitudes in order to solve algebraic equations, but he does not a lot of examples with references to everyday life. Kästner introduced the negative numbers in a practical way. At the beginning, some examples with reference to everyday life are introduced. Then, paragraphs follow in which the four basic operations with negative numbers are exlained. He used concrete numbers instead of letters as Wolff did in his algebra chapter. Euler used concrete numbers for his explanation of negative numbers, too. While Wolff only referred to "magnitudes", Kästner once and Euler several times spoke of "numbers". Another observation is that Wolff treated negative numbers subordinatedly, while this topic is dealt with independently in the textbooks by Kästner and Euler.

For the German part, there was a common notion of the interpretation of negative numbers, namely as debts. This is the same as in earlier times (cf.). Nowadays this example is still popular and often used for the explanation of negative numbers.

Euler pointed out the difference between arithmetic operations and algebraic signs of plus and minus quite illustratively. First, he explained how to deal with the arithmetic operations. After that, he introduced plus and minus as well as algebraic signs as description of positive and negative numbers. Kästner and Wolff did not make this difference clear, which was criticized by Spaun in his work.

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By making reference to the German textbooks, we can see the development concerning the treatment of the negative numbers during the 18th century. This topic was detached from its treatment in the context of algebraic equations. Negative numbers became an independent part, either within the arithmetic or in the algebra chapter. At the same time, the authors gave a concrete definition of opposed magnitudes. In order to illustrate negative numbers, Kästner gave a lot of examples from everyday life, like assets and debts. Euler defined the negative numbers as part of the integers, and illustrated them with the help of a number line.

4.1.4 Comparison and Some Results

There are some interesting observations regarding the treatment of negative magnitudes in the considered French and German textbooks. In the French textbooks, negative numbers were treated within the algebra part, which was not always the case in the German books. At the beginning of the 18th century, Wolff treated negative numbers within the algebra. In the middle of the 18th century, however, Kästner explained negative numbers at the beginning of his *Anfangsgründe* within the arithmetic chapter. Although Euler treated the negative numbers in his textbook on algebra, he labeled negative numbers explicitly as “numbers”. Also in Kästner’s textbook, the terminology “number” is once used instead of magnitudes. In contrast, the “number”-terminology is never used by the French authors.

There are commonly accepted interpretations of negative numbers, such as possessions and debts and opposite directions, which are widely employed in both French and German textbooks. Overall, the difference between the signs + and – once as operations and once as algebraic properties of quantities is not explicitly addressed; it is completely missing in La Caille since negative numbers are only defined in the context of polynomials there.

In practice, a span from complete acceptance of negative numbers as solutions of problems (especially when those are originally formulated with no references to real life) to no acceptance (that is, the hypothesis of the problem should be reformulated), passing by a limited acceptance (that is, provided that one can link to these negative numbers an interpretation that reconnects them with reality) existed.

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4.2 Geometry: the Pythagorean Theorem

4.2.1 Introduction

The case study on the Pythagorean theorem suits to give an insight into how geometry is treated in the French and German textbooks from the 18th century. In Heath's edition of Euclid's *Elements* the Pythagorean theorem is formulated as follows: "In right-angled triangles the square in the side subtending the right angle is equal to the squares on the sides containing the right angle" (Euclid 1908, Book I, Proposition 47, p. 349).

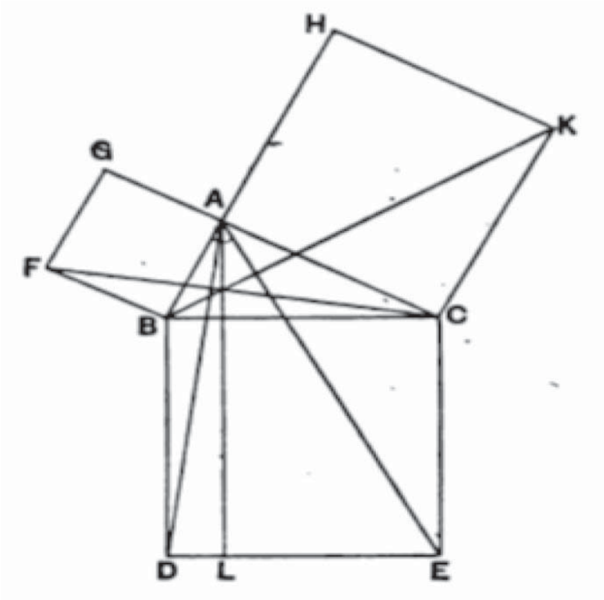

Illustration 10: Euclid 1908, p. 349

In order to prove this theorem,[25] we consider the right-angled triangle *ABC* as in the above diagram. We construct the square *BDEC* on *BC*, the square *AGFB* on *AB*, and the square *ACKH* on *AC*. Then we draw *AL* from *A* parallel to *BD* (and *CE*), and the segments *AD* and *CF*. The angles *BAC* and *BAG* are right angles. Since the line *AB* is straight, it follows that *A* lays between *G* and *C*. Thus, *BAC* and *BAG* are equal to two right angles, and *CA* forms a straight line with *AG*. In the same way, *BA* form a straight line with *AH*. Because the (right) angle *DBC* is equal to the (right) angle *FBA*, and let the same angle *ABC* be added to each, the whole angle *DBA* is equal to the whole angle *FBC*. Then we can show that the sides *AB* and *BD* are respectively equal to the sides *FB* and *BC*, because *DB* is equal to *BC*, and *FB* to *BA*. The angles *ABD* and *FBC* are equal, too. Therefore, the base *AD* is equal to the base *FC*, and the trian-

25 For the full proof, cf. Euclid 1908, pp. 349 ff.

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gle *ABD* is equal to the triangle *FBC*. The parallelogram *BL* is twice the size of the triangle *ABD*, because they have the same base *BD* and are in the same parallels *BD* and *AL*. For the same reason, the square *AGFB* is twice the size of the triangle *FBC*. Since double the amount of two equals are equal to one another, the parallelogram *BL* is also equal to the square *AGFB*. Similarly, one can show that the parallelogram *CL* is equal to the square *ACKH* by joining *AE* and *BK*. Finally, the whole square *BDEC* is equal to the two squares *AGFB* and *ACKH*, that is, the square described on *BC* is equal to the squares described on *BA* and *AC*.

We remark that we did not use any algebraic equations like $a^2 + b^2 = c^2$. We rather proved the theorem with the help of properties of angles, lines, and equal areas. We want to find out whether during the 18th century this theorem and its proof were handled in a uniform style in the *Cours* and *Anfangsgründe* textbooks. More precisely, we consider the approaches in the textbooks by Bélidor, La Caille, Camus, Bézout, and Bossut for the French part, and in the textbooks by Wolff, Kästner, Segner, Karsten, Clemm, and Klügel for the German part.

4.2.2 France

In the middle of the geometry part (Part IV), Bélidor introduced the Pythagorean theorem in Euclidean terms: "In a right-angled triangle *ABC*, the square on the hypotenuse *AC* is equal to the sum of the squares of the two other sides" (cf. Bélidor 1725, p. 202). He provided three proofs and two corollaries. The first is written in an algebraical style, meaning that at the beginning of the proof, Bélidor denoted certain segments with the letters *a*, *b*, *c*, *x*, he derived some inferences (sometimes written twice with letters and segments names), and finally he translated the conclusion into a geometrical language. There, he basically drew the hypotenuse height and used the so obtained similitude between the two triangles and the big one in order to infer some proportions, and afterwards translated the problem into a quadratic equations in more than one unknown. A short calculation with these equations leads to the conclusion. Bélidor considered the second proof as the most beautiful since it only supposes a statement on equal triangles (namely, that two triangles are equal when each side is equal). This proof is three times longer than the first one and fully written in geometrical language. It involves parallelograms, but unfortunately it seems to be corrupted and no diagram is available. The third proof is the geometrical one from the *Elements* (which Bélidor himself also remarked). Interestingly, in the corollaries, Bélidor explained to his students how he was going to use this theorem in the following: given two sides (or their squares) of a right-angled triangle, one can find the third one (or its square). He also observed that the segment perpendicular to the hypotenuse is the mean proportional

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between the segments into which the hypotenuse is divided. This was a porism to Proposition 8 in Book VI of the *Elements* and is nowadays known as the right triangle altitude theorem or as the geometric mean theorem. This provides another useful consequence of the theorem: given the hypotenuse and the two segments, one can find the other sides of the triangle. Besides this, no numerical example is given.

In the geometry part of the first volume of La Caille's textbook, in the sections concerning triangles, we cannot find any statement about the Pythagorean theorem. In fact, he did not mention the theorem as such at all. An important application of the Pythagorean theorem is to determine the sides of right-angled triangles. In La Caille's textbookwe find something similar in the trigonometry part (cf. La Caille 1741-1750, vol. 1, pp. 343-348). There, La Caille firstly gave the law of sines and briefly explained how to substitute other trigonometric functions in it. The table for "solving the right-angled triangles" follows: there, he provided the formula for the measure of all the sides and angles of a right-angled triangle, given one side and another side or one (non-right) angle. The table lists nine possible combinations and two to three formulas for each. Afterwards, La Caille generalized the procedure to non-right-angled triangles. No proof and no numerical example are given.

In the second volume on geometry, while dealing with ratios of similar figures involving triangles, Camus gave a slightly modified enunciation of the Pythagorean theorem ("If three squares *BG*, *AH*, *AO* form with their sides a right-angled triangle *BAC* and, having drawn a perpendicular *AD* from the right angle on the hypotenuse, we extend it until *F* through the square *BG* drawn on the hypotenuse, we will have *BF*=*AH* and *CF*=*AO* so that the square *BG* of the hypotenuse will be equal to the sum *AH*+*AO* of the two squares drawn on the sides *AB*, *AC* of the right angle", cf. Camus 1749-1752, vol. 2, p. 250). Camus' proof is different from the one in the *Elements* that we quoted above, though in the same geometrical style. Basically, Camus prolonged the two sides of the square on the hypotenuse that are perpendicular to the hypotenuse until they meet the prolongations of the sides of the two other squares opposite to the triangle; the same holds for the height on the hypotenuse. This enabled him to split the square on the hypotenuse in two parts that were identified in many passages with the appropriated triangles, parallelograms, and finally squares. In the first corollary, Camus described two classical proportionality relations (cf. below, the second and third points of Bézout's statement) and, in the second corollary, a property of inscribed right-angled triangles. No numerical example is given, as well as no hint on how a student could apply the theorem in practice.

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Bézout dealt with this topic virtually in the same way in both textbooks for the navy and for the artillery. We consider therefore the earliest presentation in the second volume on geometry in the navy textbook. In the subsection on similar triangles, we find the following three-folded proposition: "If, from the right angle *A* of a right-angled triangle *BAC*, we draw a perpendicular *AD* on the opposite side *BC* (that is called hypotenuse): 1. the two triangles *ADB*, *ADC* are similar to one another and to the triangle *BAC*; 2. the perpendicular *AD* is mean proportional between the two parts *BD* and *DC* of the hypotenuse; 3. each side *AB* or *AC* of the right angle is mean proportional between the hypotenuse and the correspondent segment *BD* or *DC* (cf. Bézout 1764-1769, vol. 2, p. 73). We remark that none of the above points corresponds the Pythagorean theorem as we have stated it and that our version can be found nowhere else. Actually, the first point of the above proposition is in the *Elements* (more precisely, it is Proposition 8 in Book VI), the second point is the right triangle altitude theorem, and the third point is a well-known property. Both the second and the third points can be derived from the first one and, indeed, are equivalent to the Pythagorean theorem. Bézout's proof of the first point is in substance the same as the one that we have presented above, in the Introduction, and he used the same geometrical framework and language. In our opinion, Bézout chose not to give a statement like the Pythagorean theorem because its proof is extremely articulated (even if it involves a relatively small number of results). Rather, he split the issue into three statements that have a more or less straight relation to the Pythagorean theorem. Indeed, Bézout put this theorem in another context than the *Elements*: he preferred a presentation that makes calculations possible. Nevertheless, Bézout gave no further instruction on how to concretely apply or use the proposition.

As with La Caille, we cannot find any statement about the Pythagorean theorem in the sections concerning triangles of the third volume of Bossut's textbook. Actually, we can find a section entitled "Resolution of Right-Angled Triangles" in the trigonometry part (cf. Bossut 1772-1775, vol. 3, pp. 292-299). There, two propositions enable to determine the sides and the angles of a right-angled triangle, given two sides, or one side and one (non-right) angle, respectively. The proofs have a similar structure and basically involve the tables of sines (that Bossut gave in a former section) and what we nowadays call the law of sines. For the first proposition, Bossut provided two numerical, well-detailed examples.

4.2.3 Germany

In his *Anfangsgründe* and his *Auszug*, Wolff proved the Pythagorean Theorem in the same way as Euclid did (cf. Wolff 1775, vol. 1, pp. 187 ff., and Wolff 1737, pp. 122

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ff.). In both textbooks, we can find the unquestioned remark that this theorem is named after its originator Pythagoras. In his *Anfangsgründe*, Kästner also proved the theorem as Euclid did (cf. Kästner 1800, pp. 218 ff.). Even the construction is the same as Euclid used in his *Elements*. Kästner mentioned that the theorem is named after his alleged originator Pythagoras. This statement is more critical than Wolff's. Beyond, Kästner referred to further literature regarding the origin of the theorem. After the theorem and its proof, Kästner gave two appendices: firstly, the inverse theorem and, secondly, how to construct a square of the same size as the sum (and the difference respectively) of two other squares. We have noticed that the notation of the angles was not coherent in Kästner's work. Sometimes he used the sign "Δ", sometimes the term "triangle". Although Kästner did not use any algebraic equation, we can find the term $BCq = ABq + ACq$ where q signifies "square", which is a geometrical notation because of the straight lines *BC, AB*, and *AC*.

In Segner's *Anfangsgründe* a proof similar to Euclid's is shown (cf. Segner 1764, p. 302). Segner used proportions in order to compare the triangles and concluded that the rectangles and the squares have the same size. He did not mention Pythagoras. We can find different notations for the squares: sometimes *ABCD*, and sometimes only *AC*.

The approach of Karsten's textbooks *Lehrbegrif* and *Auszug* resembles to Kästner's explanations (cf. Karsten 1767, pp. 274 ff., and Karsten 1785, vol. 1, p. 90). Karsten showed in Euclidean style with the help of the properties of the angles that the areas of the squares have the same size. As Wolff, Karsten mentioned uncritically that the theorem is named after the founder Pythagoras. He explained the notation *Abq*: *q* is put upwards on the right of the letter which denotes the straight line (cf. Karsten, 1767, p. 275). He used *q* as an abbreviation for the geometrical construction of a square and not as an algebraic sign. Further, Karsten used different notations for the polygons (*ABCD, AC, ABq*).

Clemm started in his textbook *Erste Gründe* with the legend around Pythagoras and the discovery of the theorem (cf. Clemm 1777a, p. 414). These historical remarks seem to be used for purposes of entertainment, because they are not critical. Here, Clemm also explained the terms hypotenuse and cathetus. His proof of the theorem is similar to Euclid's, but with a different notation: there, small numbers are used instead of the three capital letters of the sides of the triangle in order to mark an angle. Clemm is the first author of those we studied who used this terminology. In his textbook *Mathematisches Lehrbuch* we find the same proof of the theorem as in the textbook *Erste Gründe* (cf. Clemm 1777b, p. 207 ff.). In the *Mathematische Lehrbuch*, Clemm used the notation $AB^2 + BC^2 = AC^2$ for the first time (cf. Clemm 1777b, p. 208). He

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used also other notations for the square: *ABCD, AC* (for the diagonal), *Abq, Ab^q*. Beyond that, we find critical remarks on Pythagoras as the alleged originator of the theorem (cf. Clemm 1777b, p. 209). Clemm also remarked that there is another proof of the theorem – the 24th known proof – which originates from differential calculus and was found by professor Kies in 1766.

Klügel is the only author who gave in his *Anfangsgründe* a proof different from Euclid's. He wrote that Euclid's proof is the most common one and that he did not want to repeat it (Cf. Klügel 1792, p. 103). In Klügel's approach, two parallelograms on the sides *AB* and *AC* of the triangle are used. These parallelograms are transformed into one on the hypotenuse *BC*. Klügel is the only author who used the term "*Kathete*" (cathetus). Sometimes he used "∠" as sign for the angle, sometimes not.

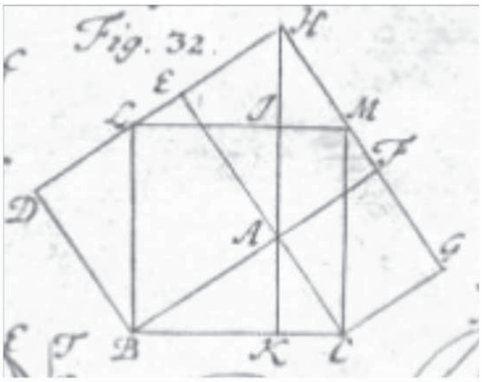

Illustration 11: Klügel 1792, fig. 32

4.2.4 Comparison and Some Results

According to our analysis of the above French textbooks, the treatment of the Pythagorean theorem was not standard. Some authors gave the statement in geometrical terms and proved it geometrically, using a different proof as the one in Euclid's *Elements*, like Camus, or proved it in part geometrically and in part algebraically, like Bélidor. Bézout's treatment is completely geometrical, and he gave some statements that are equivalent to, but more simple than the Pythagorean theorem. Other authors, like La Caille and Bossut, did not integrate the theorem at all, but rather a trigonometrical way to achieve a similar application for right-angled triangles.

With regard to the German textbooks, we showed that the authors stood in the tradition of Euclid. They treated the Pythagorean Theorem within the geometry chapters and proved the theorem geometrically. Klügel is the only author who gave an alternative proof because the other one was common. Nevertheless, there were also proofs of the

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theorem based on differential calculus, as seen in Clemm's remark. In accord with the general topics order in these textbooks – namely the explanation of arithmetic and geometry right at the beginning, it was not possible for the author to anticipate differential calculus. This is also an explanation of the fact that the authors did not use the algebraic equation $a^2 + b^2 = c^2$. Clemm is the only one of the considered author who used this notation, but only subordinatedly, in his geometrical proof.

Interesting are the different notations. First, we find various notations for a square, namely *ABCD, CS, ABq,* AB^q, AB^2. Second, there existed different markings for angles. Usually, the angles where named with the help of the three points of the triangle. For example, the angle *ABC* means that *A* and *C* are the sides and that the angle is at the point *B*. Some authors also used the sign "∠", but it must be taken into consideration that it was maybe impossible for the printers to print this sign because it was not part of the set of available printing letters. However, we observed that the authors who used this sign did not use it exclusively, but switched between the different notations. These aspects show that there was no uniform notation for squares and angles during the 18th century. In particular, small letters were not used for the sides of polygons. Instead, we find the two capital letters corresponding to the extremes of the considered segment.

4.3 Ballistics

4.3.1 Introduction

In the following, we give a brief insight into the treatment of ballistics in France and Germany in the 18th century. For the French part, we remark that, among our selection, only Bélidor and Bézout dealt with this topic. For the German part, our main reference is the textbook *Mathematische Anfangsgründe* by Kästner. We choose this textbook because it was common and very influential during the 18th century. We also briefly compare it with the textbooks by Wolff, Clemm, Karsten, and Struensee.

4.3.2 France

At the end of Part XIII in Bélidor's textbook, we find five problems on the metals in a cannonball, on the diameter of cannonballs and of the cylinder that contains the powder, on cannons' length, and on the number of cannonballs in a stack (cf. Bélidor 1725, pp. 469-490). Anyway, Bélidor mainly dealt with ballistics in the subsequent Part XIV (cf. Bélidor 1725, pp. 491-542). The approximately fifty pages of this book are divided into three chapters: the first is on the shock of bodies, the second on the motion of thrown bodies, and the third on the theory and praxis of bomb throwing. Bélidor's aim was to provide trusted rules on this topic using mathematics and physics. He presented

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concerns about the precision of throwing bombs and about the sequence of actions that a bomber should perform. The first two chapters, as Bélidor himself admitted in the short foreword to this book (cf. Bélidor 1725, p. 491), do not directly concern ballistics, but rather deal with basics and provide the principles that the reader needs. The third chapter is divided into nineteen propositions with a lot of corollaries and some definitions and questions. In the first meaningful theorem, it is shown that the trajectory of a projectile is a parabola, assuming that the air makes no resistance. Bélidor used geometrical ways of arguing in the proof, for instance the parabola is defined through a proportion that involves the square of the ordinates and the abscissas. From there, there follows a series of problems that are tightly linked to the practice of a bomber: in general, it is about, given some parameters of throwing (height, angles, charge), finding the remaining parameters in order for the bomb to reach the target. Afterwards, Bélidor explained how to construct a "universal instrument" which enables to throw bombs even at targets that lie on the same altitude as the cannon. This is a kind of three dimensional goniometer that helps to calculate the above mentioned parameters. Finally, we find two theorems on throwing two projectiles at the same time that are useful when one does not have the universal instrument at hand. These theorems are proved using trigonometry. Their applications are clearly explained for the bombers. Two problems on the force of projectiles end the book. Summing up, Bélidor's treatment of ballistics is concise and its main prospective is to provide applications and reduce the mathematics in it to a manageable amount. Nevertheless, these two last aspects are mutually dependent: in order to keep the mathematics reasonably simple, Bélidor overlooked the case in which projectiles are subject to air resistance, which is essential in practice. A far more extended treatment of ballistics can be found in the *Le Bombardier François* (cf. Bélidor 1731).

The treatment of ballistics is integrated in both Bézout's textbooks, for the navy (cf. Bézout 1764-1769, vol. 5, pp. 149-172) and for the artillery (cf. Bézout 1770-1772, vol. 4, pp. 72-88, 128-197, and 438-469), with several substantial differences. As we expect, the artillery textbook is the most comprehensive one. Nevertheless, the mathematical immersion is not always the most advanced there, which is also not surprising since we recall that especially the parts on pure mathematics were generally shorter and less in-depth in the textbooks for the artillery. In the navy textbook, the ballistics part is in the middle of the book on mechanics. It deals with the nature and properties of the trajectory of a projectile in non-resistant environments and with ricochets. The case of the free fall from the mast of a ship is also considered. At the very end, Bézout added a few paragraphs on the trajectory of a projectile in resistant environments. These are written in small characters, and we recall that a peculiarity of Bézout's text-

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books is that he distinguished the size of the characters depending on the public that he was addressing: normal sized characters for all students, small characters for the advanced. While justifying his statements, Bézout preferred to make reference to common sense reasoning at first, before starting with calculations. Nevertheless, this part is deeply soaked with calculations and higher mathematics: not only algebra and trigonometry, but also differential and integral calculus are employed. The appliction to a bomber's practice that we have observed in Bélidor's textbook is hardly dealt with.

In the volume on mechanics of the artillery textbook, the topics concerning ballistics are split into three parts. Firstly, in the section on the general application of the mechanic principles to motion and equilibrium, we find a handful of pages that deal with the speed that projectiles achieve by an elastic condensed fluid such as the air or the burning powder. Moreover, it is dealt with the drawing back of weapons. The treatment of the behavior of weapons in non-resistant environments of the navy textbook (except for the case of the free fall from the mast of a ship and the last part in small characters) is proposed again in the artillery textbook at the end for the section on mechanic applications. Nevertheless, it is more extensively treated and augmented by appropriated examples. The remaining part of this section deals with the core of topics that a bomber should know. Indeed, Bézout dealt herein with the motion of projectiles in resistant environments. Since he needed to take a lot of factors into account, such as the changing of the air resistance depending on the height that the cannonball reaches, he had to use some approximation methods. He also presented several tables with data gathered during various experiments. Finally, he developed an equation that describes the motion of projectiles in the air with a satisfactory degree of accuracy. Afterwards, Bézout added some remarks in small characters on border cases, namely up to which point his approximations comply with experience (including some examples and a comparison table between the calculated and observed ranges of a thrown projectile), and on air resistance. The third part about ballistics is an appendix at the end of the volume, where Bézout dealt more specifically with the motion of projectiles in resistant environments. There, he commented on the preceding paragraphs, mainly discussing another, more complicated method that provides a more precise approximation to reality. He also wanted to determine more rigorously the curve which represents the trajectory of a projectile in a uniformly dense environment. As in the case of the navy textbooks, all justifications are given in the end by calculations and only seldom through non-mathematical reasoning. Since the introductory part is shorter compared to the amount of the whole treatment of ballistics, justifications by common sense reasoning are necessarily used less throughout the textbook in comparison to the comlicated integrals or long equations employed by Bézout. A permanent feature of

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both textbooks is the deep concern with details, which requires an extremely deepened level of mathematical knowledge.

4.3.3 Germany

Artillery in Kästner's Mathematische Anfangsgründe

Kästner treated topics relevant for the artillery in his *Anfangsgründe der angewandten Mathematik: Astronomie, Geographie, Chronologie und Gnomonik* ([4]1792). While Kästner treated other mathematical disciplines extensively, he only devoted seventeen pages (cf. Kästner 1792, pp. 554-570) to this issue. There is another difference in comparison to the aforementioned French textbooks in the artillery chapter besides its extent. The mathematical disciplines in Kästner's *Anfangsgründe* were all treated in the same style. In each chapter, the different items are headed with titles such as "explanation", "theorem", "proof", "task" and "solution". Nevertheless, in the chapter on artillery as well as on fortification and architecture, the items are instead numbered and do not have any specific name. Kästner gave a reason for this different handling: "These sciences [namely artillery, fortification, and architecture] are not completely mathematical: the texture of the powder and architectural stuff are part of the useful natural history and chemical physics".[26] However, for Kästner it was important that his "short remarks contain at least as much as every scholar should know in order not to look ridiculous often" .[27]

On the seventeen pages on the artillery, one finds 24 items. There are no related illustrations in the copper plates found at the end of the textbook. The chapter begins with the definition: "The artillery furnishes information on the use of the powder and the instrument for which it is used both for pyrotechnics, which serves in war, and for air".[28] In items 2 to 6, remarks on the gunpowder, on its texture, on the mixture ratio, on its effect and production are listed. This information concerns chemical physics. Items 7 and 8 are devoted to the ballistics trajectory. In item 9 to 14, Kästner described the different weapons and their composition. In items 15 and 16, one finds some mathematical calculations. Further remarks on the composition and usage of the different

26 Translated by Desirée Kröger. Original quote in Kästner 1792a, preface to the third edition, p. vii: "*Von diesen Geschäfften [nämlich Artillerie, Fortifikation und Baukunst] ist nicht Alles mathematisch: Beschaffenheit des Pulvers, und Bauzeugs, gehören zur brauchbaren Naturgeschichte und chemischen Physik*".

27 Translated by Desirée Kröger. Original quote in: Kästner 1792a, p. vi: "*meine kurzen Nachrichten* […] *wenigstens so viel [enthalten] als jeder Gelehrte von diesen Dingen wissen muß um nicht oft lächerlich zu werden*".

28 Translated by Desirée Kröger. Original quote in: Kästner 1792a, p. 554: "*Die Artillerie ertheilet Nachrichten von dem Gebrauche des Pulvers und der Werkzeuge mit denen es sowohl zu Ernstfeuerwerken, welche im Kriege dienen, als zur Luft angewandt wird*".

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weapons are mentioned in items 17 to 20. The tracking of a bullet is described in items 21 to 23. In item 24, Kästner gave further information and references to other authors.

Artillery is treated without any calculus, because the latter is firstly handled in the following third part of the *Anfangsgründe*. Even if Kästner usually presupposed knowledge which had already been treated in previous parts of the textbooks, there are here some mathematical calculations with reference to geometry. In item 15, Kästner gave some examples on how to calculate the diameter of a bullet of known weight and material. In item 16, further calculations on the diameter of a bullet are integrated.

In item 8, Kästner described the trajectory as a parabola, only regarding the forces on the bullet, the gravitation, and the weight of the bullet, but the actual shape of the trajectory depends on the angle of the shot in relation to the horizon and on the speed of the shot. If the air drag is furthermore considered, the trajectory is not a parabola, but another curve. This curve can be analyzed only with the help of higher mechanics. In item 23, Kästner repeated that the air drag may not be disregarded. In items 21 and 22, he treated the range of the shots which is closely linked to the trajectory and the tangent angle. To item 21 belongs a table which contains the range of a shot from different cannons. In item 22, it is written that the furthest shot is achieved if a bullet is fired at an angle of 45 degrees. Kästner also provided a historical remark on the theory of the trajectory as parabola by stating that this theory is ascribed to Galilei. Overall, the remarks are not justified through mathematical calculations.

Beyond the table in item 21, there is another table which belongs to item 19 where Kästner treated the differences of "*Stücken*".[29] In this table, we find a description of the differences of these cannons regarding the length of the tube, the weight of the iron bullet which is used for the shot, the weight of the bullet, the weight of the whole "*Stück*", and the amounts of constables, henchmen, and horses needed. It seems that these results are based on mathematical calculations, but Kästner did not add the corresponding calculation.

Throughout the whole chapter, Kästner referred to a lot of authors who wrote about the artillery. In item 24, he explicitly recommends textbooks which are devoted exclusively to artillery for further information. Among others, the works by Michael Mieth, Johann Sigmund Buchner, Casimir Simienowicz, Leonhard Euler, Henning Friedrich Reichsgraf von Graevenitz, Papacino d'Antoni (translated by Georg Friedrich von Tempelhof), and Étienne Bézout are referred to.

29 Cannons.

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A brief comparison with other German textbooks

In the second volume of Wolff's *Anfangsgründe* topics concerning the artillery are treated on 78 pages (Wolff 1775, vol. 2, pp. 515-592) distributed into 184 items. It is a more extensive treatment than in Kästner's textbooks.

In the extensive textbooks *Lehrbegrif* and *Anfangsgründe* Wenceslaus Johann Gustav Karsten did not treat the artillery, but he did in his compact *Auszug aus den Anfangsgründen und dem Lehrbegriffe der mathematischen Wissenschaften*. The remarks and the extent are similar to Kästner's. Karsten dealt with the artillery on 22 pages and formulated 28 items (Karsten 1785, vol. 2, pp. 409-430).

Heinrich Wilhelm Clemm treated the artillery in his *Mathematisches Lehrbuch* on two and a half pages (350-352) in 9 items (§§ 846-854).

We notice that the remarks in Kästner's *Anfangsgründe* are less extensive than in Wolff's textbook. In later mathematical textbooks, there are even less remarks on the science of the artillery. A reason for this change might be that at the beginning of the 18th century, when Wolff wrote his *Anfangsgründe*, no textbook on the artillery already existed. This was different during the time when Kästner wrote his textbook. In 1760, Karl August Struensee published a whole textbook on the science of the artillery. He was a teacher of philosophy and mathematics at the Ritterakademie in Liegnitz (cf. ADB 1893, vol. 36, p. 662). His *Anfangsgründe der Artillerie* counts 498 pages and 29 tables in the third edition from 1788. It seems that this textbook was quite popular because it was published in three editions. In the first part of his textbook, Struensee described the composition of the gunpowder and its effects over 44 pages (21-64) and 44 items (§§ 12-55). Kästner put instead this information in 5 items (§§ 2-6). In the second part of the textbook, Struensee treated the use of the gunpowder and also described different cannons over 333 pages (pp. 65-397) and 277 items (§§ 56-332). In this part, also remarks on the speed of the bullets, ballistics trajectory, firing range, and air drag are given. This information can be also found in Kästner's textbook, but in a briefer version. In the third part, Struensee handled the use of gunpowers for pyrotechnics over 66 pages (pp. 398-463) and 63 items (§§ 333-395). Kästner did not treat this part at all in his *Anfangsgründe.*

Artillery at the University of Göttingen

Today, it is not possible to study military sciences at German universities. The situation was quite different in the 18th century. The University of Göttingen was one of the leading universities in the 18th century. Kästner was the second professor of mathemat-

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ics and physics there, so it might be interesting to take a look at the lecture catalogs[30] of the University of Göttingen, where various lectures on mathematical disciplines are announced. Subjects concerning the artillery were taught since the 1760s. The first evidence can be found in the winter semester (WS) 1761/62. The Oberbaukommissar Johann Michael Müller lectured on fortification, tactics, and artillery, and the Magister Johann Paul Eberhard just on the artillery. From summer semester (SS) 1762 until SS 1795, Eberhard regularly delivered lectures on the artillery, often in combination with pyrotechnics.[31] In the lecture catalogs of SS 1769, one learns that Eberhard lectured about the artillery on the basis of Struensee's textbook.[32]

Also, there were others who gave lectures on subjects concerning the artillery, for instance a certain architect Heine in SS 1789 (lectures in fortification in general, fortification in the field, and artillery, all together) and the Ingenieurmajor Gotthard Christoph Müller on artillery and mines in WS 1790/91. Müller also gave some lectures on military sciences on the basis of his own textbook[33] from SS 1791 until WS 1792/93, in WS 1793/94, and in WS 1794/95. From SS 1795 until SS 1803, he also gave lectures on military sciences, if there was a demand. Because Müller's textbook also contains an extensive chapter on artillery, it seems probable that he also taught it.

There were also some other lectures on the military sciences, for instance the introduction to the military sciences by Magister Georg Johann Ebell on the basis of the textbook by Mauvillon[34] from WS 1784/85 until WS 1787/88.

Although artillery was also taught at the University of Göttingen, there is no evidence that the lecturers used Kästner's textbooks as a basis. On the contrary, they used textbooks which were only devoted to military sciences in general or especially to artillery.

The situation at the University of Göttingen cannot be generalized for all German universities. Göttingen was a big and modern university in contrast to other German universities (cf. Paulsen 1960, p. 11). So, it seems plausible that there was no such extensive offer of the mathematical sciences at other universities.

30 They can also be found in the *Göttingische Anzeigen von gelehrten Sachen*.

31 SS 1763, WS 1763/64, SS 1765, WS 1765/66, WS 1766/67, SS 1768, SS 1769-WS 1771/72, WS 1772/73, WS 1773/74, SS 1774-SS 1776, SS 1777-SS 1780, WS 1781/82-SS 1783, SS 1784-SS 1787, WS 1788/89, WS 1789/90-SS 1795. In WS 1779/80 including a "*Minierkunst*" (arts of mines).

32 This might be *Anfangsgründe der Artillerie* (1760, ²1769, ³1788).

33 This might be *Über militärische Encyclopädie für verschiedene Stände.* Göttingen, 1791.

34 This might be Mauvillon, Jakob: *Einleitung in die sämmtlichen militärischen Wissenschaften für junge Leute, die bestimmt sind, als Offizier bey der Infanterie und Kavallerie zu dienen*. Braunschweig, Waisenhaus-Buchhandlung, 1783 and 1784.

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4.3.4 Comparison and Some Results

Our analysis shows that ballistics received a different treatment in Germany and France. One reason for this could be the different positions of the artillery. In France, where more specific schools and therefore an intended audience were present, the people were more interested in the military sciences than in Germany. While Kästner wrote his *Anfangsgründe* mainly for the use at universities, Bézout wrote them for specialized schools.

Summing up, it can be said that Kästner treated artillery not scientifically: it seems that he wanted to give a short introduction and some basic knowledge for conversation on a superficial level. For those who want to learn more about the artillery, he referred to other scientific textbooks on this topic. So, Kästner's remarks on artillery may be considered as a basis for further and deeper studies on this subject. In contrast, Bélidor's and especially Bézout's treatments were highly technical and intended for officers who needed to put them into practice. Thelast one was, indeed, one of the textbooks referred to by Kästner.

In Germany, specialized schools for those military corps like the artillery were founded in the 18th century (cf. Kühn 1987, p. 71). Some textbooks on ballistics were written by the teachers of those specialized schools, as it can be seen at the example of Struensee. As a consequence, less contents on this topic were found in the comprehensive *Anfangsgründe* which had an encyclopedic character.

4.4 Fortification

4.4.1 Introduction

In the 18th century, fortification belonged to the overall domain of the mathematical sciences, in particular to applied mathematics. Wolff provided in his *Lexicon* the following description: fortification aims at designing a territory in such a way that the enemy cannot invade the fortress and that a small number of people could defense themselves against a higher number of people (cf. Wolff 1716, col. 647).

In the case study on fortification, we deal with the question of how it is treated in mathematical textbooks in the 18th century. We focus on the following questions:

- How comprehensive are the contents on fortification?
- Which focus laid the authors? Are definitions or skills more important?
- What are the mathematical aspects within fortification?

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4.4.2 France

Especially in the late 18th century, the French authors that we have taken into account did not include the topic of fortification – also called military architecture – in their textbooks on the mathematical sciences. Indeed, it is not even mentioned in La Caille's, Camus', Bézout's, and Bossut's textbooks.

Bélidor's treatment is partially an exception. In Part VIII of his *Nouveau Cours de Mathématique* we find a section devoted to the "Problems of Trigonometry Applicable to Fortification" (cf. Bélidor 1725, pp. 358-363). There, Bélidor briefly (over only five and a half pages) proposed two problems, namely about calculating angles and segments of an inaccessible figure, and two corollaries. The problems are formulated in a purely trigonometrical style, while the corollaries add that these problems can be useful to find the distance of some objects (to be read: "targets") in a besieged town. No further practical suggestions are given. We can explain this exception in the 18th century panorama of the textbook on the mathematical sciences by taking into account that Bélidor was a professional military engineer.

Actually, there do exist some textbooks on fortification in this period, but these are independent volumes that did not belong to the kind of literature with *Cours de(s) Mathématique(s)* as title. Bélidor himself, due to his profession, wrote a whole book on military architecture in 1729, entitled *La Science des Ingénieurs dans la Conduite des Travaux de Fortification et d'Architecture Civile*. In six chapters, it is explained how to apply the principles of mechanics to buildings, the mechanics of vault, basic knowledge about construction materials, the construction of military and civil buildings, the decoration of buildings, and cost estimation. As it was already the case for the *Nouveau Cours de Mathématique* the style is spoken-like and mathematics is only treated in a superficial way.

Nevertheless, many textbooks on fortification were written before the 18th century. Renown french architects were, for instance, Antoine Deville (1596-1657), Adam Freitag (1602-1664), Blaise-François de Pagan (1604-1665), and Sébastien Vauban (1633-1707), who were all active during the 17th century. The latest author in this tradition was François Blondel (1618-1686), who wrote, among others, treatises and textbooks on civil architecture. His *Nouvelle Manière de Fortifier les Places* (1683) is composed of two parts ("*discours*"), which contain no mathematics, and of a short appendix with mathematical contents, and some diagrams (see, for instance, Illustration 12) and tables. This treatise had been constantly reprinted until 1711 and translated into German in 1686 by Johann Hoffmann.

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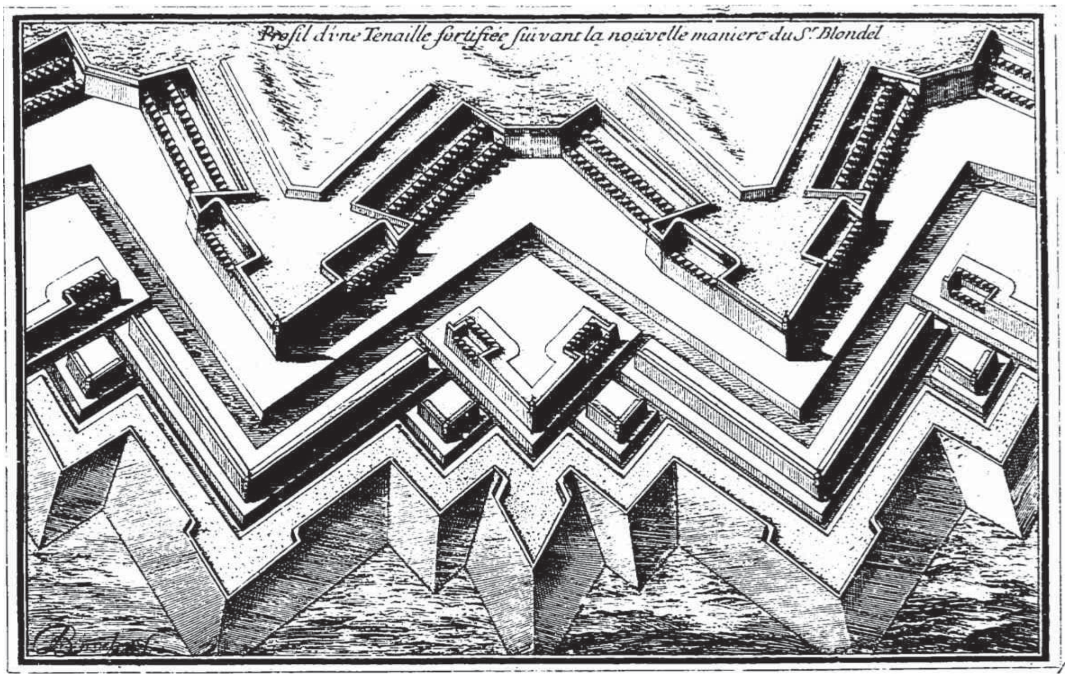

Illustration 12: Diagram from Blondel's Nouvelle Manière de Fortifier les Places *(1683), p. 94*

4.4.3 Germany

Commonly, the German textbook authors treated fortification together with artillery and civil architecture. Nowadays, fortification is no mathematical topic any more. At the University of Göttingen, a variety of lectures on this topic was offered. Not only professors, but also architects and engineers lectured on fortification. The following scholars announced lectures on fortification in the summer semester 1759: Professor Mayer, Commissarius Müller, and Architect Eberhard (cf. GGA 1759, 41. St., p. 369). Under this aspect, the University of Göttingen was an exception: normally, universities did not have enough capacity in order to offer such special lectures, and fortification was taught at knight academies (cf. Lind 1992, p. 5)

We have found some textbooks in German on fortification which were written by teachers, officers, and engineers: Karl August von Struensee (1735-1804), Andreas Böhm (1720-1790), Johann Rudolph Fäsch (1680-1749), and Gotthard Christoph Müller[35] (?-1803). Böhm was a professor for mathematics and philosophy at the University of Gießen (cf. ADB 1876, vol. 3, pp. 61 f.). In 1776, he published his two-volume textbook *Gründliche Anleitung zur Kriegs-Baukunst*. Fäsch, author of *Kurtze jedoch grund- und deutliche Anfangs-Gründe zu der Fortification* (1725), was an engineer major (cf. Fäsch 1725, title page), but he also was an architect, a theorist of architectu-

35 The date of birth is unknown; cf. Pütter 1788, p. 142.

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re, and in military service (cf. NDB 1959, vol. 4, pp. 741 ff.). Müller was an engineer major and teacher for mathematics and military sciences at the University of Göttingen (cf. Müller 1796, title page).

In our study on fortification, we will concentrate on the mathematical *Anfangsgründe*. Firstly, we will analyze Kästner's *Mathematische Anfangsgründe*. Then, we will compare the contents with other textbooks, namely the ones by Sturm, Wolff, Clemm, Karsten, and Klügel. We will also look at Struensee's textbook on fortification in order to find out which contents it has in common with mathematical textbooks.

Kästner treated fortification in his textbook *Anfangsgründe der angewandten Mathematik. Der mathematischen Anfangsgründe II. Theil, II. Abtheilung* (^{4}1792) over twelve pages and thirty passages. Two images belong to this part (cf. Illustration 13). In contrast to other topics in Kästner's *Anfangsgründe*, fortification as well as artillery and civil architecture are briefly explained.

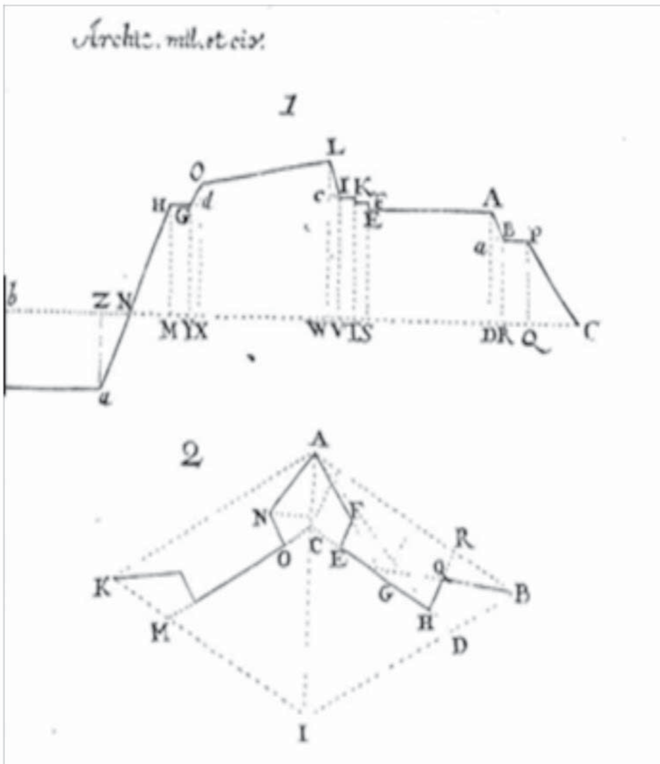

Illustration 13: Kästner 1792a, Tab. XIIII.

Kästner treated fortification in his *Anfangsgründe* because it was important for him that scholars knew the basics so that they were able to participate in conversations on this topic (cf. Kästner 1792a, preface to the third edition, p. vi). He also stated that there were enough people who were more qualified to teach fortification than him (cf.

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Kästner 1792b, preface to the first edition, without page reference). On the basis of these passages, we can assume that Kästner wanted to provide fundamental knowledge. It was not unusual that fortification was taught for the purposes of conversations (cf. Hohrath 2005, p. 113). The fact that Kästner did not renounce the treatment of fortification is an indicator of the encyclopedic character of this textbook.

Kästner stated that there is not only mathematics (geometry and mechanics) within fortification, but also knowledge on chemical physics, natural history, and customs (cf. Kästner 1792a, preface to the third edition, p. vii.). At the beginning, he defined fortification in the same manner as Wolff in his *Lexicon* did. Then, he explained some components of fortresses and referred to the illustrations. It is interesting to notice that Kästner did not only give the German, but also the French names of the components. This points at the fact that, at this time, fortification was a subject dominated by the French architects.

Kästner concentrated on regular fortifications, which means that the ground plan of the fortress is a regular polygon. He therefore needed to make reference to the definition of a polygon within the geometry chapter. In this manner, a link to mathematics within fortification was established.

In some passages, Kästner mentioned also concrete specifications. While describing the position of bastions, he suggested that they should have a distance to each other of sixty "*rheinländische Ruthen*" (about 12 feet) (cf. Kästner 1792a, p. 572). This distance derives from the range of a musket, a long gun, and is remarkable because it reveals how the building of a fortress depends on the kinds of weapons that were used. Afterwards, Kästner explained ramparts, bulwarks, moats, and their functions with many references to the images (cf. for instance Illustration 13). Literature on different types of fortification, like Leonhard Christoph Sturm's *Architectura Militaryis Hypothetico* (1720), Benjamin Hederich's *Progymnasmata Architectonica, oder Vor-Übungen in beyderley Bau-Kunst* (1757), and Albrecht Ludwig Friedrich Meister's (1724-1788) article *De Variis Architectorum Conatibus Optimam Munimenti Formam Ope Analyseos Definiendi* (1779) in the Latin journal *Commentationes Societatis Regiae Scientiarum Gottingensis* of the Göttingen Academy, was referred to. For interested readers, Kästner advised further literature: *Anfangsgsründe der Kriegsbaukunst* by Struensee, *Gründliche Anleitung zur Kriegs-Baukunst* by Böhm, and some not explicitey named articles in the *Magazin für Ingenieure und Artilleristen* (cf. Kästner 1792a, p. 583). We remark that there was a journal in German on these topics for engineers and artillerymen.

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At the beginning of the 18th century, Sturm published his textbook *Kurtzgefasste Mathesis*, in which he dealt with fortification over nine pages and by means of twenty illustrations. Sturm's textbook is segmented into so-called tables ("*Tabellen*") or rather subchapters. Six tables belong to the fortification chapter, where Sturm treated some basics, including the terminology of the elements of the ground plan and the elevation of fortresses, the ground plan of a regular fortification, the ground plan and elevation of outworks in uneven territories, redoubts, and some new kinds of fortresses. Like Kästner, Sturm did not only mention the German but also the French names of the parts of the fortress. By explaining the parts of a fortress, Sturm referred to images which are drawn in perspective (cf. Illustration 14).

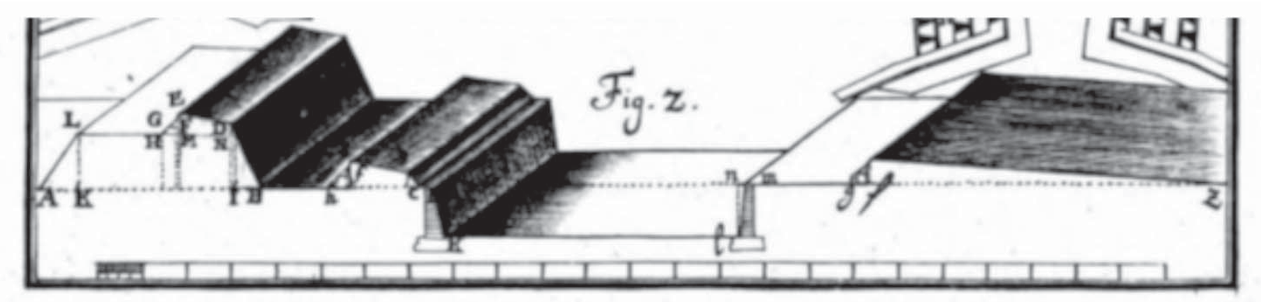

Illustration 14: Sturm 1717, first table to fortification, fig. 2

Interestingly, Sturm provided some tables which contain the length of lines and the size of angles in a fortress (cf. Illustration 15)[36]. At the end of this subchapter, Sturm described how such tables can be produced. Thereby, he emphasized the importance of geometry, trigonometry, and arithmetic for such calculations.

II. Täffelein derer vornehmsten Winckel.

		V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
Der Mittel-Puncts-Winckel.	A Z B	72	60	51. 25	45	40	36	32 44	30
Der Kehl-Winckel.	B A F	108	120	128. 35	135	140	144	147 16	150
Der Bollwercks-Punct.	H A G	60	65	73. 35	80	85	89	92 16	95
Der Winckel der Cortin mit der Streich-Linie.	H t h	24	27. 30	27. 30	27. 30	27. 30	27. 30	27. 30	27. 30
Der Winckel der Schulter mit der Streich-Linie.	H m I	90	85	85	85	85	85	85	85

Illustration 15: Sturm 1717, p. 43

36 The Roman numbers indicate the number of edges in a polygon.

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In the first volume of Sturm's *Mathesis Juvenilis* (1710-1714), the chapter on fortification is more extensive than in his *Kurtzgefasste Mathesis*. It counts 183 pages and is divided into 160 sections. Fifty-one images belong to this chapter.

At the beginning, Sturm briefly illustrated the history of fortification and different manners of fortification building. The first battlements were made of fences, and later of walls. These kinds of battlements did not suffice so that they need to be improved in the course of years. In his explanations, Sturm named the parts of a fortress using the German and the Latin terms. In the first subchapters, Sturm presented different types of fortifications and explained how to create ground plans for the different types. He explained the French, Italian, and Dutch construction ways, and those by the military architects Melder, de Ville, Reyher, Pagan, Rusenstein, Vauban, Blondel, Scheither, and Rimpler (cf. Sturm 1710/14, vol. 1, pp. 620-708). For all types, Sturm gave instructions on how to draw a fortress on the paper. He made geometric calculations of the necessary lines and angles, whereby he explained the calculation steps. He registered the results in tabular form – in the same manner as in his *Kurtzgefasste Mathesis*. This approach has two advantages: firstly, it is possible to follow gradually and understand the process of developing the tables; secondly, engineers could benefit from the tables because they did not have to calculate it anymore, but could directly use the data for specific constructions. In the third and last subchapter, Sturm expounded some parts of a fortress (outworks, breastworks) and the building of irregular fortifications. Here, the focus lays on the explanation of terms. We find only few calculations and tasks like drawing a bulwark onto an irregular polygon (cf. Sturm 1710/14, pp. 771-773).

The comparison of the two considered textbooks by Sturm shows significant differences. The treatment of fortification in the *Mathesis Juvenilis* is more extensive than in the *Kurtzgefasste Mathesis*. In the former work, which was composed primarily for students at grammar schools and which contains tasks for the different classes, Sturm laid the focus on the presentation of different types of fortification. We find numerous calculations and corresponding tables. In his textbook *Kurtzgefasste Mathesis*, which is addressed to beginners in general, Sturm was less concerned with the manners of various military architects and, instead, put the focus on the explanation of individual components of a fortress.

In the second volume of Wolff's *Anfangsgründe aller mathematischen Wissenschaften* (1775), fortification is treated over 144 page. The explanations are separated in 356 sections with 26 theorems, 61 definitions, 59 tasks, a few examples, additions, and remarks. The whole chapter is divided into five subsections:

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- on the principles of fortification;
- on different types of fortification;
- on irregular fortifications, citadels, and breastworks;
- about the real construction of the fortress;
- on the attacks and the resistance of them.

In the preface to the fortification chapter, Wolff informed the readers that he would explain the principles of fortification and the most common types (cf. Wolff 1775, vol. 2, pp. 595 ff.). He also addressed all the relevant angles and lines which are necessary for the construction of a fortress and that can be calculated with the help of geometry and trigonometry.

In the first subsection, Wolff explained several terms concenirng fortification. With 174 sections, it is the largest of the five subchapters. In the definitions ("*Erklärungen*") Wolff explained terms related to the fortress, and addressed their function. In some passages, the reader also finds dimensions of the individual components. This approach is characteristic for Wolff. At the beginning, each term or each component of a fortress and its function is explained. Wolff mentioned not only the German but also the French terms. Sometimes one can find concrete dimensions. Besides the theoretical explanations, Wolff referred to figures, so that the reader can imagine the content. In addition, Wolff pointed to previously treated contents and to the relevant paragraphs. At times, he also referred to various different types and named the respective military architect (see, for instance, the reference to Rimpler; cf. Wolff 1775, vol. 2, p. 606).

Especially the theorems ("*Lehrsätze*") aroused our attention. These do not ressemble mathematical theorems but rules. The related proofs ("*Beweise*") can be considered as justification for the theorems (cf. Illustration 16).

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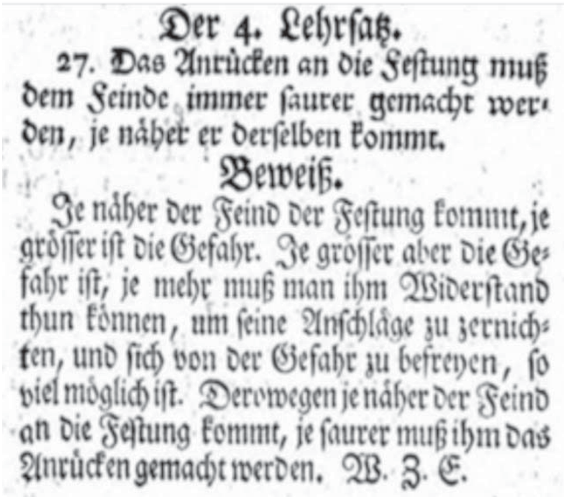


Der 4. Lehrsatz.

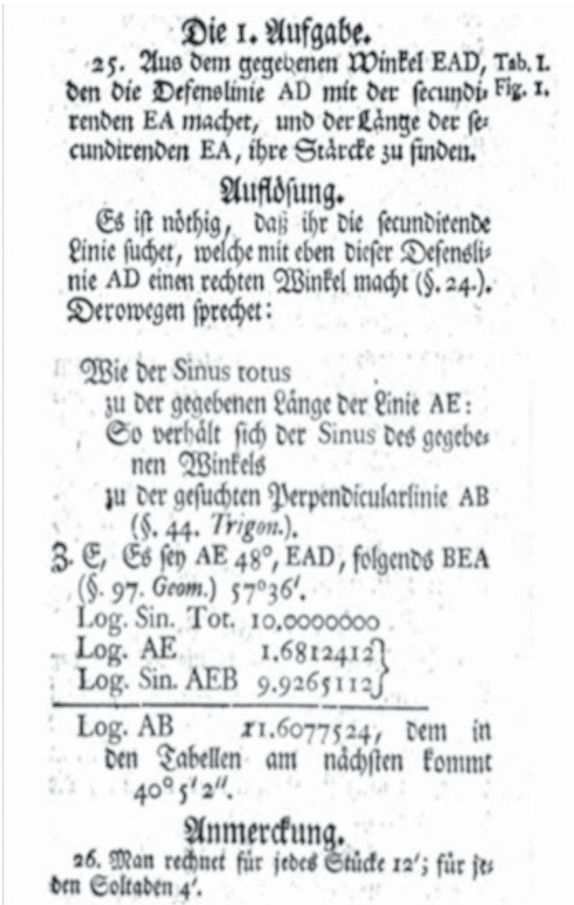
27. Das Anrücken an die Festung muß dem Feinde immer saurer gemacht werden, je näher er derselben kommt.

Beweiß.

Je näher der Feind der Festung kommt, je grösser ist die Gefahr. Je grösser aber die Gefahr ist, je mehr muß man ihm Widerstand thun können, um seine Anschläge zu zernichten, und sich von der Gefahr zu befreyen, so viel möglich ist. Derowegen je näher der Feind an die Festung kommt, je saurer muß ihm das Anrücken gemacht werden. W. Z. E.

Illustration 16: Wolff 1775, vol. 2, p. 604

Wolff did not confine himself to the imparting of facts, but also represented concrete calculations which we find in the tasks (“*Aufgaben*”) (cf. Illustration 17). Again, there are references to illustrations and paragraphs or contents from geometry and trigonometry, which are necessary for calculations for the framework of the fortification.



Die 1. Aufgabe.

25. Aus dem gegebenen Winkel EAD, den die Defenslinie AD mit der secundirenden EA machet, und der Länge der secundirenden EA, ihre Stärcke zu finden. Tab. I. Fig. I.

Auflösung.

Es ist nöthig, daß ihr die secundirende Linie suchet, welche mit eben dieser Defenslinie AD einen rechten Winkel macht (§. 24.). Derowegen sprechet:

Wie der Sinus totus
zu der gegebenen Länge der Linie AE:
So verhält sich der Sinus des gegebenen Winkels
zu der gesuchten Perpendicularlinie AB
(§. 44. *Trigon.*).

Z. E. Es sey AE 48°, EAD, folgends BEA (§. 97. *Geom.*) 57°36′.

Log. Sin. Tot. 10.0000000
Log. AE 1.6812412
Log. Sin. AEB 9.9265112

Log. AB 11.6077524, dem in den Tabellen am nächsten kommt 40° 5′ 2″.

Anmerckung.

26. Man rechnet für jedes Stücke 12′; für jeden Soltaden 4′.

Illustration 17: Wolff 1775, vol. 2, p. 603 ff.

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In the first subsection, we find occasional references to military architects like Scheither, Pagan, de Ville, Vauban, and Borgdorf. These references are related to certain components and dimensions. The specific types are presented in the second subsection for the first time. In the second subsection, Wolff covered a variety of architectural styles of fortifications (cf. Wolff 1775, vol. 2, pp. 641-686). He began with the Dutch manner, and went on to the French methods of Pagan, Blondel and Vauban. Wolff presented these because they were the best known (cf. Wolff 1775, vol. 2, p. 686.). Again, the French style of fortification dominated. For each method we find an explanation, examples and numerous tasks: for instance, how to draw angles, lines, ground plans, and other parts of a fortress after a certain manner. It appears that it came down to exercises on the paper because Wolff's explanations are quite detailed. In order to solve the tasks or to perform concrete calculations, geometric and trigonometric skills are necessary. As a result of the calculations, Wolff gave some tables, for instance showing the angles according to the Dutch manner (cf. Illustration 18)[37]. In addition to the numerous tasks, many examples are given.

Grösse der Winkel in Holländischen regulären Festungen.

Nahmen der Winkel.	IV	V	VI	VII	VIII	IX	X	XI	XII
Centri-Winkel	90°	72°	60°	51°26'	45°	40°	36°	32°44'	30°
Polygon-Winkel	90	108	120	128.34	135	140	144	147.16	150
Bollwercks-Winkel	60	72	80	85. 42	90	90	90	90	90
Kleine-Winkel FGE	15	18	20	21. 26	21. 30	25	27	28. 38	30
Schulter-Winkel	105	108	110	111.26	111.30	115	117	118.38	120

Illustration 18: Wolff 1775, vol. 2, p. 643

In the third subsection on fortification, Wolff addressed irregular fortifications. At the beginning, he explained not only irregular fortifications, but also regular ones with reference to the second subchapter, in which he had presented various types. In addition to six statements, readers will find fourteen objects on how to fortify any part of a fortress. Wolff did not only depict theoretical explanations. It seems to have been important to him that the reader could put his knowledge into practice. For this intention,

37 The Roman numbers indicate the number of edges in a polygon.

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the fourth subsection, which deals with the actual construction of a fortress, is integrated. Besides the familiar constructions and calculation tasks for certain components, Wolff also took other aspects into account. These include the costs and the time needed for the building (cf. Wolff 1775, vol. 2, pp. 713 ff.). Here, Wolff used arithmetic, namely the rule of three ("*Rule Detri*"), and referred to the corresponding paragraph in the arithmetic section. He also included the treatment of the nature of the soil and the associated requirements in his calculations (cf. Wolff 1775, vol. 2, pp. 715 ff.). In the fifth subsection, Wolff focused on possible attacks and on the defense of a fortress. This is the primary tactic during an attack. He also mentioned other issues, such as the supply of the crew with food and ammunition (cf. Wolff 1775, vol. 2, p. 722).

In Clemm's *Mathematisches Lehrbuch* fortification is part of architecture. The chapter on fortification comprises sixteen pages, 41 sections, and twelve illustrations (cf. Clemm 1777b, second part, pp. 337-350). The whole chapter is divided into five passages:

- history of fortification;
- rules of fortification and the names of the main components;
- different types of regular fortifications;
- on irregular fortifications;
- something about artillery.

After the definition of fortification, which is similar to that by Wolff, Clemm gave a short overview of the history of fortification. The reader learns that fortification had been dealt with since ancient times, and that there were different types of fortification (cf. Clemm 1777b, second part, pp. 337 ff.). Then, Clemm went on to the contemporary styles of fortifications, which is directly linked to the developments of the artillery. In the context of historical considerations, Clemm discussed the different manners of fortification building. He pointed out, among others, that the Italian, the Dutch and the French manner were leading (cf. Clemm 1777b, second part, p. 338). At this, Clemm mentioned the names Freytag, St. Julien, Blondel and Vauban. The latter three were French military architects, who Clemm referred to as "masters of the military architecture" (Clemm 1777b, second part, p. 338). One recognizes that French architects were well-respected during Clemm's times. In the end, Clemm mentioned some literature on fortification: Leonhard Christoph Sturm's *Architectura Militaryis Hypothetico-eclectica, Oder Gründliche Anleitung zu der Kriegs-Baukunst*, St. Julien's *Architectura Militaryis*, Daniel Specklin's *Architectura von Vestungen*, Johannes Faulhaber's *Ingenieurs-Schul*, Georg Berhard Bilfinger's remarks on his own style of

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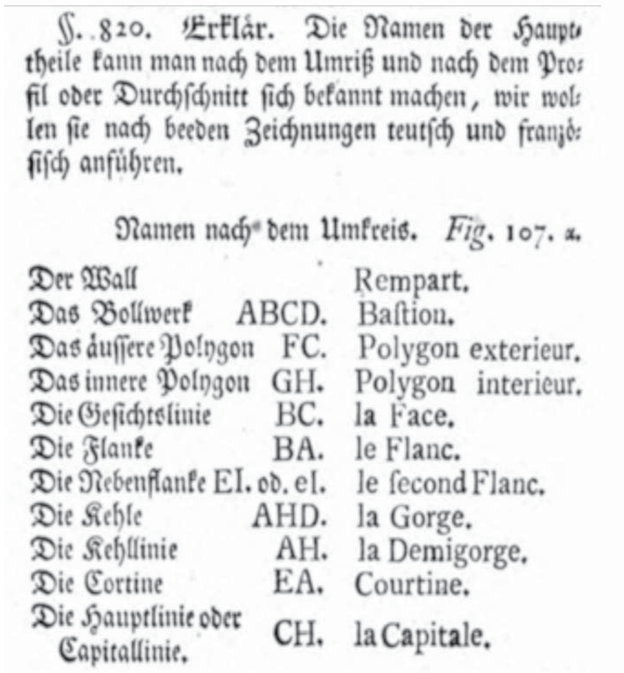
building fortifications, and Andreas Böhm's *Gründliche Anleitung zur Kriegs-Baukunst* (cf. Clemm 1777b, second part, p. 339). Beyond that, Clemm mentioned also some treatises on artillery by Casimir Simienowicz, Michael Mieth, Benjamin Robin, and Karl August von Struensee. In the second subchapter, Clemm explained some components of a fortress, both in German and French (cf. Illustration 19). Their functions are expounded with the help of illustrations.



§. 820. Erklär. Die Namen der Haupttheile kann man nach dem Umriß und nach dem Profil oder Durchschnitt sich bekannt machen, wir wollen sie nach beeden Zeichnungen teutsch und französisch anführen.

Namen nach dem Umkreis. *Fig*. 107. a.

Der Wall		Rempart.
Das Bollwerk	ABCD.	Bastion.
Das äussere Polygon	FC.	Polygon exterieur.
Das innere Polygon	GH.	Polygon interieur.
Die Gesichtslinie	BC.	la Face.
Die Flanke	BA.	le Flanc.
Die Nebenflanke	EI. od. eI.	le second Flanc.
Die Kehle	AHD.	la Gorge.
Die Kehllinie	AH.	la Demigorge.
Die Cortine	EA.	Courtine.
Die Hauptlinie oder Capitallinie.	CH.	la Capitale.

Illustration 19: Clemm 1777b, second part, p. 340

According to observations, Clemm indicated that the drawings, which were created for the construction of fortresses, are known from elementary geometry and trigonometry, namely from the discussion of polygons (cf. Clemm 1777b, second part, p. 342). The reader learns that mathematical knowledge is important for fortification.

Afterwards, Clemm presented different manners of fortification building, namely the Dutch, some French (by Pagan, Blondel, Vauban, Bélidor), and the German ones (by Bilfinger) (cf. Clemm 1777b, second part, pp. 343-349.). The various styles and their characteristics are shown in explanations/definitions ("*Erklärungen*"). In the additives, Clemm performed calculations and informed the reader about the individual systems and their vulnerabilities. Remarkable is the paragraph which is denoted as theorem ("*Lehrsatz*") (cf. Clemm 1777b, second part, pp. 348 f.). Here, Clemm described how to draw a fortress using the system of Bilfinger. It is a guided task. He presented two cases, namely, that the basic shape of the fortress is a triangle or a quadrilateral.

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The last subchapter is dedicated to irregular fortifications whereby the deviation from the regular fortifications should remain as small as possible. Clemm achieved a close connection between fortification and the artillery by adding some comments on the artillery at the end of the chapter on fortification (cf. Clemm 1777b, second part, pp. 350-352.). Here, the reader will find explanations about the artillery, guns, as well as shots. At the end, Clemm referred to further literature on the trajectory of projectiles, more particularly, to the works of Euler, Robin, Gravenitz, and Karsten.

Since Karsten's textbooks *Lehrbegrif der gesamten Mathematik* and *Anfangsgründe der mathematischen Wissenschaften* remained incomplete, we find no architectural sciences in it. In the second volume of Karsten's *Auszug aus den Anfangsgründen und dem Lehrbegriffe der mathematischen Wissenschaften* fortification is treated in the last chapter. In the preface, Karsten wrote that it was customary to provide a brief guide on the architectural sciences in mathematical textbooks, but it in not possible to present the contents completely (cf. Karsten 1785, vol. 1, preface to the first edition, pp. xii ff.). Karsten's version comprises 21 pages with 44 paragraphs, sixteen definitions, and one task. He supports his remarks with fourteen figures.

In the definitions, Karsten explained some terms like fortress, rampart, and parapet. It is remarkable that Karsten only used the German terms and not the Latin ones any more. In some passages, he also mentioned recommended dimensions (cf. for instance Karsten 1785, vol. 2, p. 426). In the explanation of the various components of a fortress, Karsten also addressed different angles. He required geometry knowledge and used terms such as "parallel" and "inward-opening angle" in the description of the parapet (cf. Karsten 1785, vol. 2, p. 431). Within the explanation of the different manners of military architecture, Karsten emphasized the benefits of geometrical knowledge (cf. Karsten 1785, vol. 2, p. 442). He did not cover the individual manners in detail, but only mentioned the names of famous military architects, namely of Freytag, Pagan, Vauban, Blondel, Cöhorn, and Rimpler. He recommended to the reader the *Anfangsgründe der Kriegsbaukunst* by Struensee as comprehensive work on military architecture (cf. Karsten 1785, vol. 2, p. 442). In addition to the explanations, Karsten also formulated a task, namely how to fortify a military camp or an area that is occupied by an army (cf. Karsten 1785, vol. 2, p. 435). In its resolution, Karsten referred to some illustrations and gave concrete measures for the establishment of the individual components. In this context, the difference between regular and irregular fortifications and the various styles of military architecture are mentioned (cf. Karsten 1785, vol. 2, pp. 435-438).

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Klügel treated fortification in his textbook *Anfangsgründe der praktischen Mechanik, der bürgerlichen Baukunst und der Kriegsbaukunst* (1784). The corresponding contents that Klügel used simultaneously for his lectures are abstracted without any changes from the third volume of his *Encyklopädie oder zusammenhängender Vortrag der gemeinnützigsten Kenntnisse* (1784) (cf. Klügel 1784, p. iii). The chapter on fortification in Klügel's *Anfangsgründe* counts 64 pages with 155 passages and is divided into four subchapters: on the artillery; on the fortification of cities and areas; on the attack and the defense of fortresses; on the field fortification.

At the beginning, Klügel defined fortification. At the end, we find a register with sixteen German and French books on artillery and fortification. To the chapter belongs a copper plate with five illustrations. Klügel wanted to explain as much about fortification as necessary, so that civil people could understand news on this topic (cf. Klügel 1784, p. 231). Thus, he concentrated on basic knowledge, namely explanations of components of a fortress. He did not use Latin, but only German terms. He also explained the difference between regular and irregular fortifications. He mentioned different manners of fortification building, namely those by Vauban, Cöhorn, and Rimpler (cf. Klügel 1784, pp. 268-271).

Klügel also looked at some principles from the artillery. He justified this integration with the fact that fortification is closely linked to artillery (cf. Klügel 1784, p. 231). It is interesting that, for him, the artillery is part of fortification. In the most considered textbooks, artillery formed a separate chapter. In the part about artillery, Klügel at first considered the composition of gunpowder and stated the exact mixing ratios as well as the manufacturing process (cf. Klügel 1784, p. 232). Then, information about different guns and their structure, cannons and their charge, gunpowder, balls and caliber, and various types of shots followed (cf. Klügel 1784, pp. 232-250). In these explanations, we find references to mechanics and geometry, as well as to further literature, for example to the *Magazin für Ingenieure und Artilleristen*. In addition, there are tables that show certain relations, for instance the length of calibers in relation to the weight of the balls in certain guns (cf. Illustration 20).

Klügel's explanations are more extensive than those in the other textbooks from the second half of the 18th century that we studied. He clarified the close relationship between fortification and artillery, and thereby justified the inclusion of a subchapter on artillery. Klügel explained not only the main terms and components of a fortress, but also their functions.

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	Länge in Kalibern.	Schwere der Kugeln.	
Ganze Karthaunen	18	48	Pf.
Dreyviertel Karthaunen	20	36	—
Halbe Karthaunen	22—24	24	—
Viertel Karthaunen	24	12	—
Achtel Karthaunen	27	6	—
Regimentsstücke	14—18	3	—
Ganze Feldschlangen	30	18	—
Halbe Feldschlangen	36	9	—
Viertel Feldschlangen	34	4—5	—
Falkaunen	27	5—6	—
Falkonets	35—36	2—3	—
Halbe Falkonets	38	1	—
Serpentinels	40	$\frac{1}{2}$	—

Illustration 20: Klügel 1784, p. 236

Struensee's *Anfangsgründe der Kriegsbaukunst* were quite common since the last third of the 18th century. A lot of authors we considered in our studies referred to Struensee's textbooks. He studied theology at first, then mathematics and philosophy at the University of Halle (cf. ADB 1893, vol. 36, pp. 661-665). From 1757 to 1771, he was a teacher for mathematics and philosophy at the knight academy in Liegnitz. Because of the lack of suitable textbooks on military topics, he decided to publish his *Anfangsgründe der Artillerie* (1760, ^{2}1769, ^{3}1788). Another comprehensive textbook of him is *Anfangsgründe der Kriegsbaukunst* in three volumes (1771-1774, ^{2}1786-1789).

Because we concentrate on mathematical textbooks which were used at universities, we will only give a short insight into the first edition of Struensee's textbook *Anfangsgründe der Kriegsbaukunst* in order to compare the contents with them of the mathematical *Anfangsgründe*. The knight academy is an institution different from university. While various sciences were taught at a university and formed different occupational groups, the aim of a knight academy was the formation of nobles and people for the military service. Accordingly, we can expect that the contents in Struensee's textbook are not only more detailed, but also more application-oriented. Struensee's textbook has got three volumes:

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- *Anfangsgründe der Kriegsbaukunst. Erster Theil, so von der Befestigungskunst im Felde handelt* (1771), on field fortification;
- *Anfangsgründe der Kriegsbaukunst. Zweyter Theil, darin von der Beschaffenheit der eigentlichen Festungen gehandelt wird* (1773), on the characteristics of common fortresses;
- *Anfangsgründe der Kriegsbaukunst. Dritter und letzter Theil, so von dem Angrif und der Vertheidigung der Festungen handelt* (1774), on the attacks and the defense of fortresses.

The three volumes together count 1678 pages with 1550 paragraphs and 99 copper plates – this is a remarkable difference to other textbooks. From the preface to the first volume of Struensee's *Anfangsgründe,* we can draw interesting conclusions on his view of fortification. First, Struensee wrote that there already were many works on fortification, so that he had initially hesitated to publish his work (cf. Struensee 1771, p.)(2^{r} f.). Most of the works have been written by engineers, that is, by experienced people. Struensee himself had no practical experience, but was a teacher of the mathematical sciences at the knight academy in Liegnitz. However, he did not know any complete and accurate textbook that satisfied his requirements. Struensee claimed that it had become common that the military sciences were only marginally considered in mathematical textbooks. The explanations were limited to artificial words and the usual fortresses (cf Struensee 1771, p.)(3^{r}.). He saw, however, due to the current war in his time, the need to write about field fortification, the attacks on fortresses, and their defenses, because these topics were only occasionally presented in the common textbooks, but were important for the young officers, his primary target group (cf. Struensee 1771, p.)(3^{v}).

We have seen that fortification was considered as a mathematical science in the 18th century. However, Struensee named fortification as an engineering science (cf. Struensee 1771, p. 1). This is an indication for its separation from the mathematical sciences and the simultaneous establishment of engineering. At the beginning of his remarks, Struensee gave an introduction to fortification, in which he discussed, among other things, the history of fortification and indicated further works on this topic.

The chapters of Struensee's textbook are divided into main pieces and sections. At the margin of the text, bullet points or short phrases are noted that characterize the relative content. Struensee also integrated mathematical calculations, for instance when calculating a parapet and the associated angles and lines (cf. Struensee 1771, pp. 71 ff.). He decided not to separately deal with the various styles of military architects, but rather

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picked them up locally in individual parts again and again. Until the tenth chapter of the second volume he did not deal with the different building styles of renowned military architects.

In Struensee's textbook, we find the description of the components of a fortress, including their respective function – just as in the mathematical textbooks. The second volume of his *Anfangsgründe* includes, among other things, statements on walls and outworks. Here, readers also find instructions on how to draw these components on the paper, distinguishing between regular and irregular fortifications. In contrast to the mathematical textbooks we studied, Struensee was interested in an effective implementing of knowledge. He included a lot of comments about the staking and the materials necessary for the construction, also considering how many workers were needed (cf. Struensee 1771, §§ 130-185). The following main piece contains more remarks on the additional supply of a fortress, for example with ammunition and food. (cf. Struensee 1773, §§ 463-484).

4.4.4 Comparison and Some Results

For the French *Cours*, we remarked that fortification was usually not included in the mathematical sciences; it was rather a topic for itself.

In contrast, our study shows that the German mathematical *Anfangsgründe* contained fortification as an independent topic. Sometimes it was linked to artillery. The explanations on fortification were less comprehensive than on other topics. Kästner and Klügel wrote explicitly that they only wanted to give as much knowledge about fortification as to achieve a general understanding, also for conversational purposes. This is the only explicit justification of the authors of the *Anfangsgründe* for teaching fortification. That this topic was still integrated in mathematical textbooks, testifies to the encyclopedic nature of these textbooks. The situation is different in Struensee's *Anfangsgründe der Kriegsbaukunst*, which is a special textbook on military architecture and was addressed to a different group of students, namely *inter alia* later officers.

One reason for the reduction of the contents on fortification in the mathematical textbooks in the second half of the 18th century may be that, by then, there existed enough professionals who taught fortification and published specific textbooks. This is an indication that fortification gradually emancipated itself from the mathematical sciences and became independent.

The authors of the *Anfangsgründe* concentrated on the definitions and explanations of the main components and their functions for a fortress. Struensee commented, as well as Klügel, on field fortification. He also addressed the actual building of a fortress.

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Such an explanation – only shorter – we also find in Wolff's textbook. The other authors of textbooks concentrated, if they did at all, on the construction of fortresses just on the paper. These authors emphasized the importance of geometry and trigonometry for the field of fortification. With the help of these disciplines the calculation of the lines and the angles of the components of a fortress was possible. Some authors presented tables with the results of concrete calculations.

Fortification became more important during the 18th century. This is visible in the increase of number of specific textbooks and articles which the authors of the *Anfangsgründe* referred to. By means of the historical remarks in Clemm's textbooks, we learn that France had an important role in the field of fortification. Most of the authors we considered did not only use the German but also the French names of the components of a fortress. Above that, the most of the presented building styles came from French military architects.

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5 Conclusion

In this work, we tried to provide an overview of the topics taught under the label "mathematical sciences" during the 18th century in France and Germany. To this aim, we have analyzed a representative sample of the new comprehensive textbooks that were written in this period.

These textbooks reacted to the general attitude of how knowledge was understood after the Enlightenment and the spread of encyclopedias. One overall feature is that they were aimed at a better understanding on the side of the student, therefore offering a lot of examples and applications of the displayed theoretical contents. From our analysis emerged that the conception of these textbooks, both in France and in Germany, is tightly related to the respective educational context. In France, these textbooks were written for and used in military schools specific for the navy, the military engineers, or the artillery; in Germany, they were generally conceived for universities and higher schools. This might explain the fact that the French textbooks present much more technical features than the German ones, especially concerning the amount of the mathematical knowledge employed there.

We paid particular attention to the question of which disciplines where under the umbrella of the mathematical sciences. Quite at the opposite in comparison to nowadays, a certain amount of what we now consider applied mathematics was included in the textbooks designated for the mathematics teaching. Some of the considered textbooks provided, interestingly, not only the necessary basis for the mathematical sciences, since the students supposed to read them were beginners, but also some higher mathematics such as differential and integral calculus and algebra.

More in detail, we remarked that these kind of French and German textbooks were not always comparable due to smaller or greater dissimilarities, and this in several varied domains. Indeed, the case study on negative numbers shows that, in France, their treatment is quite homogeneous and is always dealt with within the algebra part. In contrast, this does not hold for Germany, where some textbooks include negative numbers in the algebra part and some in the arithmetic part. The case study on the Pythagorean theorem also displays dissimilarities. This time, the German authors provided a uniform presentation, more linked to traditional Euclidean geometry, while the French sometimes favored a more calculatory points of view, namely involving trigonometry. The case study on ballistics displays further dissimilarities: in French textbooks, especially in those from the second half of the 18th century, this topic is dealt with including much more mathematics, and of a higher level, than in Germany. Again,

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this can be explained recalling the different educational contexts. Finally, the case study on fortification reveals the greatest dissimilarities. Due to the increasing specialization of knowledge, fortification had simply disappeared from the French textbooks on the mathematical sciences. However, this process had not yet taken place in Germany and we still find traces of this topic in the German textbooks.

Finally, a few words about the tradition of these textbooks in the following centuries: Bezout's textbooks were still used during the 18th century. Despite the French Revolution, that marked a break also in the teaching institutions in France, there was a certain continuity in contents. Other textbooks that came in use after the revolution are those by Lacroix, Bossut's successor as *examinateur* (at the École Polytechnique) and professor at the Collège de Quatre Nations. Nevertheless, they were not presented under a unitary title (*Traité Elémentaire d'Arithmétique* in 1797, *Traité Elémentaire de Trigonométrie* in 1798, *Elémens de Géométrie* in 1799, *Complément des Elémens d'Algèbre* in 1800, *Traité Elémentaire de Calcul Differéntiel et du Calcul Intégral* in 1802). This continuity probably occurred because France had already experience the separation between pure and applied mathematics at an early time so that, in the available textbooks, the most of the applied mathematical sciences, like fortification, had already been deleted. In contrast to the situation in France, the *Anfangsgründe* were no longer used in 19th century Germany. The reason might be that through the Prussian educational reform in 1810 new textbooks were needed and they were more specific and not so widespread as the *Anfangsgründe*. After the 18th century, we can no more find a textbook which contains the variety of the mathematical sciences as before. This is also connected to the fact that the classification of the mathematical sciences changed in the 19th century: for instance, some topics became independent from mathematics and are today listed among physics.

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6 Appendix: Comparison of the Contents on Ballistics in Kästner's and Bézout's Textbooks

Kästner, Abraham Gotthelf: *Anfangsgründe der angewandten Mathematik. Der mathematischen Anfangsgründe* II. Theil. II. Abtheilung. Astronomie, Geographie, Chronologie und Gnomonik. Göttingen, Vandenhoeck und Ruprecht, [4]1792. "Von der Artillerie", S. 554-570.		Bézout, Etienne: *Cours de Mathématiques à l'usage du Corps Royal de l'Artillerie*, 1788, Paris, volume IV, Mechanics and hydrostatic applications, paragraphs 409-422, 468-549, and 823-854, pp. 72-88, 128-197, and 438-469.	
Definition of ballistics	§ 1		
Mixture of the powder	§§ 2-3		
Samples of the powder	§ 4		
Impact at inflammation	§§ 5-6		
Trajectory	§§ 7-8	Trajectory	§§ 470-471
Description of different weapons ("*Stücken*" and "*Mörser*")	§§ 9-10		
Description of different parts and instruments of the weapons (gun barrel, instruments for loading, caliber)	§§ 11-13		
Kind of bullets in different weapons	§ 14		
Calculation of the diameter of a bullet with known weight	§§ 15-16		
Composition of a "*Mörser*"	§ 17		
Munition of a "*Mörser*"	§ 18		
Differences of "*Stücken*"	§ 19		
Length of the "*Stück*" in connection with the inflammation	§ 20		
Direction of "*Stücken*" and "*Mörser*"; about the tracking	§ 21		
About the parabola trajectory	§ 22	About the parabola trajectory	§§ 472-482
About the air drag	§ 23	About the air drag	§§ 496-547, 823-852
Further literature	§ 24	Further literature	§§ 853-854

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7 Appendix: Short Biographies

In the following, in alphabetical order, the most important biographical facts about the authors of the selection of textbooks that we have analyzed will be presented. Circumstances that concern their didactic roles will be stressed particularly.

7.1 Bernard Forest de Bélidor (1698-1761)

Illustration 21: Portrait of Bélidor from the title page of the Architecture Hydraulique*, vol. 1, part 2 (1750)*

Bélidor had a double career as an army officer and a military engineer. When Bélidor's talents for practical mathematics came to the attention of the Duc d'Orléans, the latter discouraged him from entering holy orders and arranged his appointment as professor for mathematics in La Fère. He had the position at this artillery school between 1720 and 1738. He was also in charge of some administrative duties: since 1758, he was inspector of the Arsenal in Paris and, since 1759, general inspector of the mines. After an interval of active duty abroad, Bélidor settled in Paris where, in 1756, he was elected to the Académie des Sciences. Since 1726, he was also a member of the Royal Society. For more information about Bélidor's biography, cf. Fourcy 1761.

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7.2 Etienne Bézout (1739-1783)

Illustration 22: Portrait of Bézout

Bézout was the son of a magistrate, but he surprisingly did not follow the same career. In 1763, he was appointed as *examinateur* of the mathematical sciences for the naval officers, the position being offered to him by the Duc de Choiseul. On this occasion, Bézout was also required to write a textbook about the mathematical sciences for the candidates. At Camus' death in 1768, he took up similar duties for the artillery, with an interruption between 1772 and 1779. He was part of the board of examiners for these military corps until the very end of his life. Bézout was moreover in charge of the teaching of physics at the École des Élèves for the artillery, at first between 1762 and 1765 as Abbé Nollet's assistant and then, in 1770-1772, as his substitute. He applied for the professor position, but Monge, who at that time was 24, was instead retained. Thanks to his textbooks for these military schools, Bézout became appreciated and gained an indisputable authority. Furthermore, his research works in mathematics were quickly recognized by the Académie des Sciences, which he entered in 1758. He was also a member of the Académie Royale de Marine in Brest. For detailed information about Bézout's biography, cf. Alfonsi 2005, Alfonsi 2011, and Condorcet 1783.

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7.3 Charles Bossut (1730-1814)

Illustration 23: Portrait of Bossut

Bossut studied at the Jesuit College in Lyon, together with Joseph Lalande and Jean Étienne Montucla. He took minor ecclesiastical orders until 1792. He had many well-known mentors, like d'Alembert, Clairaut, and Camus. Especially thanks to the recommendation of Camus, Bossut obtained the position of professor for mathematics at the École du Génie in 1752 in Mézières, where he remained until 1768. There, Monge became his assistant for the hydraulic class. In this time span, he tried to improve the level of Camus' lectures, but without much success since the latter was still the *examinateur*. Bossut wanted to be innovative, in particular in mixed mathematical subjects like perspective, shadow theory, infinitesimal calculus, dynamics, and hydrodynamics. At Camus' death in 1768, he became *examinateur* for the military engineers. During the French Revolution he lost this position, but, thanks to Monge, he became *examinateur* for the École Polytechnique. Bossut was also a scholar. Thanks to d'Alembert, he was *correspondant* of the Académie des Sciences wit 23 and became a member in 1768. He also entered the academies of Bologna, Saint Petersburg, and Turin. For more information about Bossut's biography, cf. Mathias 1932.

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7.4 Nicolas-Louis de La Caille (1713-1762)

Illustration 24: Portrait of La Caille

La Caille was an astronomer and a famous professor. After having graduated in theology at the Collège de Navarre, he took deacon's order. From 1739 to 1762, he taught mathematics at the Collège Mazarin, in the position previously held by Varignon. In 1741, in recognition of his work on the meridian and his resolution of the controversy over the shape of the earth, he entered the Académie des Sciences in Paris. Moreover, he was a member of the academies in Berlin, Saint Petersburg, Bologna, Stockholm, and Göttingen. For more information about La Caille's biography, cf. Fourcy 1762.

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7.5 Charles Étienne Louis Camus (1699-1768)

Illustration 25: Commemorative plaque for Camus at his birthplace in Crézy-la-Chapelle

Camus, a former student of Pierre Varignon, had academic, teaching, and administrative positions throughout his life. He was mainly an *académicien*, active both as an administrator and as a scientist. He first established his reputation in mathematics by winning a prize of the Académie des Sciences in 1727, and in the same year he entered this institution. In 1730, he was appointed to the Academy of Architecture, where he also taught geometry. He was also a member of the Academy of the Navy. He was moreover the predecessor of Bézout and Bossut as *examinateur* for the artillery and for the military engineers, respectively. For more information about Camus' biography, cf. Fourcy 1768.

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7.6 Heinrich Wilhelm Clemm (1727-1775)

Illustration 26: Necrology of Heinrich Wilhelm Clemm in the abbey church in Tübingen

Heinrich Wilhelm Clemm was born in Hohen-Asperg on 13th December, 1725. In 1743, he started his studies of philosophy and mathematics at the abbey in Tübingen and received the master`s degree in 1745. From 1745 to 1748, Clemm studied theology in Tübingen. From 1750 to 1752, he worked as a lecturer at the abbey in Tübingen. He lectured on philosophy, theology, Hebraic, and mathematics. After an educational journey through Germany and a short time as vicar at the court chapel in Stuttgart, Clemm was a lecturer and a priest in Bebenhausen from 1754 to 1761. Subsequently, he was a teacher for mathematics at the Gymnasium in Stuttgart until 1764, and a librarian until 1767. From 1764 until his death, he was teacher for theology in Tübingen. Clemm died on 27th July, 1775.

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7.7 Wenceslaus Johann Gustav Karsten (1732-1787)

Illustration 27: Portrait of Karsten

Wenceslaus Johann Gustav Karsten was born in Neubrandenburg on 15th December, 1732. He studied theology and philosophy at the universities in Rostock and Jena from 1750 to 1754. Beyond his studies, he attended private lectures on mathematics and received the master's degree in 1755. After this time, he taught as a private lecturer of mathematics at the University of Rostock. He obtained the professorship of logics in Rostock in 1758. In 1760, Karsten went to the University of Bützow as a professor for logics, mathematics, and physics. From 1778 to 1787, Karsten was an ordinary professor for mathematics and physics at the University of Halle. He was the successor of Segner and the forerunner of Klügel there. Karsten died in Halle on 17th April, 1787.

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7.8 Abraham Gotthelf Kästner (1719-1800)

Illustration 28: Portrait of Kästner

Abraham Gotthelf Kästner was born in Leipzig on 27th September, 1719. On his 12th birthday, he started his studies at the University of Leipzig and received his master's degree in 1736. From 1739 on, Kästner worked as a private lecturer at the University of Leipzig and became extraordinary professor for mathematics in Leipzig in 1746. Ten years later he was appointed ordinary professor for mathematics and physics at the University of Göttingen as successor of Segner. Kästner remained in Göttingen until his death on 20th June, 1800. Kästner was the teacher of Klügel, Lichtenberg, and Hindenburg. He was a member of numerous scientific societies and interested in mathematical problems, for instance the problems of the parallels. For further reeding on Kästner, cf. Baasner 1991.

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7.9 Georg Simon Klügel (1739-1812)

Illustration 29: Portrait of Klügel

Georg Simon Klügel was born in Hamburg on 19th August, 1739. He studied theology and mathematics at the university of Göttingen. In 1763, he published his well-known dissertation *Conatuum Praecipuorum Theoriam Parallelarum Demonstrandi Recensio*. Klügel's teacher and advisor Kästner motivated and supported him to write about this topic. From 1767 to 1788, Klügel was ordinary professor for mathematics at the University of Helmstedt, and from 1788 until his death ordinary professor for mathematics and physics and the successor of Karsten at the University of Halle. Klügel died in Halle on 4th August, 1812.

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7.10 Johann Andreas von Segner (1704-1777)

Illustration 30: Portrait of Segner

Johann Andreas von Segner was born in Preßburg (Hungary) on 9th October, 1704. From 1725 to 1730, he studied medicine, mathematics, and physics at the University of Jena. Then, he worked as a practitioner in Preßburg and in Debrezin. In 1732, he received the master's degree at the University of Jena and worked there as a private lecturer until 1733. In 1733, he was appointed as extraordinary professor for mathematics and physics at the University of Jena. In 1735, he was appointed as ordinary professor for mathematics and physics at the University of Göttingen. He was the forerunner of Kästner. Since 1736, Segner was also ordinary professor for medicine in Göttingen. From 1755 to 1777, he was ordinary professor for mathematics and physics at the University of Halle. He was the successor of Wolff and the forerunner of Karsten. Segner died in Halle on 5th Obtober, 1777.

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7.11 Johann Christoph Sturm (1635-1703)

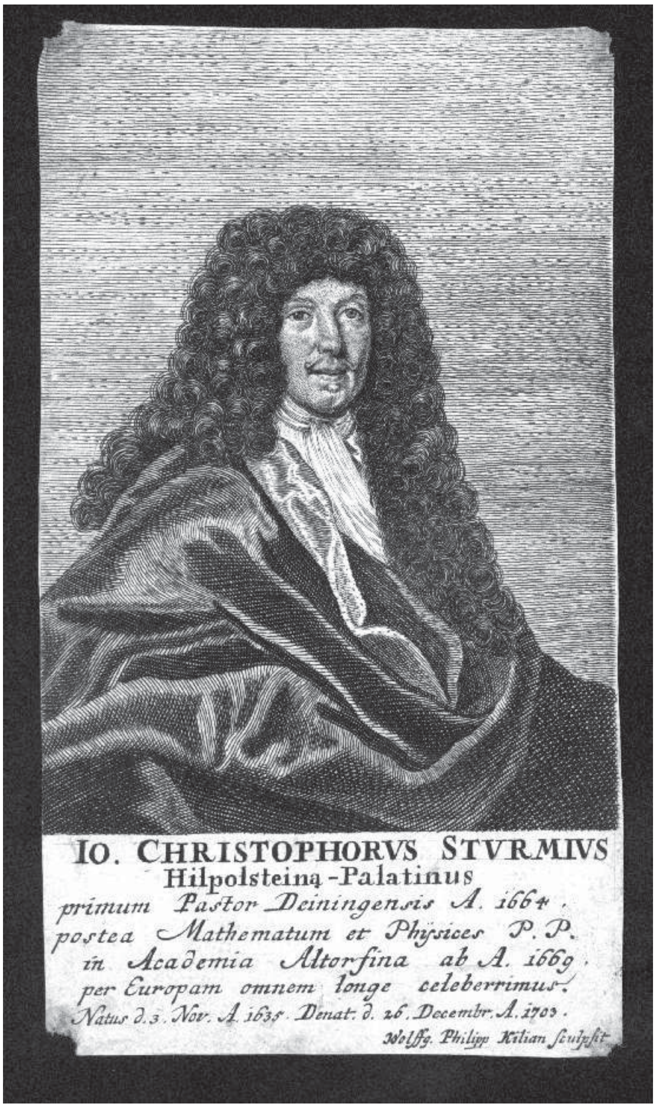

Illustration 31: Portrait of Sturm

Johann Christoph Sturm was born in Hilpoltstein on 3rd November, 1635. He studied mathematics, physics, and theology at the University of Jena form 1656 to 1662. After this time, he worked as a private teacher in Nürnberg. Form 1664 to 1669, he worked as a pastor in Deiningen. He was a professor for mathematics and physics at the University of Altdorf from 1669 to 1703. Sturm died in Altdorf on 26th December, 1703.

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7.12 Christian Wolff (1679-1754)

Abbildung 32: Portrait of Wolff

Christian Wolff was born in Breslau on 24th January, 1679. In 1699, he started his studies in theology and mathematics at the University of Jena. He got qualified as professor in 1703. Wolff was ordinary professor for mathematics and physics at the University of Halle from 1707 to 1723. Then he was dismissed and changed to the University of Marburg. In 1740, Wolff returned to the University of Halle. He was the forerunner of Segner. Wolff died in Halle on 9th April, 1754.

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SURIREY DE SAINT-RÉMY, Pierre (1745): *Mémoires d'artillerie*, three volumes, 3rd edition augmented by G. LEBLOND. Paris: Jombert.

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8.2 List of the *Anfangsgründe der Mathematik*

CLEMM, Heinrich Wilhelm (1759, ^{2}1769, ^{3}1777): *Erste Gründe aller mathematischen Wissenschaften*. Stuttgart: Mezler.

CLEMM, Heinrich Wilhelm (^{2}1768, ^{3}1777): *Mathematisches Lehrbuch, oder vollständiger Auszug aus alles so wohl zur reinen als angewandten Mathematik gehörigen Wissenschaften, nebst einem Anhang oder kurzen Entwurf der Naturgeschichte und Experimentalphysik*. Stuttgart: Mezler.

EULER, Leonhard (1770): *Vollständige Anleitung zur Algebra,* two volumes. Petersburg: Kaiserliche Akademie der Wissenschaften.

ERXLEBEN, Johann Christian Polycarp (1772, ^{2}1777, ^{3}1784, ^{4}1787, ^{5}1791, ^{6}1794): *Anfangsgründe der Naturlehre*. Göttingen: Dieterich.

HÜBSCH, Johann Georg Gotthelf (1745): *Anfangs-Gruende der Rechen-Kunst in Bruechen, welche der Pfoertnischen Jugend in der letzten Classe vorgetragen*. Naumburg.

KÄSTNER, Abraham Gotthelf (^{1}1758-1769): *Mathematische Anfangsgründe*, first edition in six volumes, last edition in ten volumes. Göttingen: Vandenhoeck and Ruprecht. In the following, a list of the single volumes can be found:

[1] KÄSTNER, Abraham Gotthelf (1758, ^{2}1763, ^{3}1774, ^{4}1786, ^{5}1792, ^{6}1800): *Anfangsgründe der Arithmetik, Geometrie, ebenen und sphärischen Trigonometrie, und Perspectiv. Der mathematischen Anfangsgründe ersten Theils erste Abtheilung*. Göttingen: Vandenhoeck and Ruprecht.

[2] KÄSTNER, Abraham Gotthelf (1786, ^{2}1801): *Fortsetzung der Rechenkunst in Anwendungen auf mancherley Geschäffte. Der mathematischen Anfangsgründe erster Theil zweyte Abtheilung*. Göttingen: Vandenhoeck and Ruprecht.

[3] KÄSTNER, Abraham Gotthelf (1790): *Geometrische Abhandlungen. Erste Sammlung. Anwendungen der Geometrie und ebenen Trigonometrie. Der mathematischen Anfangsgründe ersten Theils dritte Abtheilung.* Göttingen: Vandenhoeck and Ruprecht.

[4] KÄSTNER, Abraham Gotthelf (1791): *Geometrische Abhandlungen. Zweyte Sammlung. Anwendungen der Geometrie und ebenen Trigonometrie. Der mathematischen Anfangsgründe ersten Theils vierte Abtheilung.* Göttingen: Vandenhoeck and Ruprecht.

[5] KÄSTNER, Abraham Gotthelf (1759, ^{2}1765, ^{3}1780, ^{4}1792): *Anfangsgründe der angewandten Mathematik. Mechanische und optische Wissenschaften. Der mathematischen Anfangsgründe zweyten Theils erste Abtheilung*. Göttingen: Vandenhoeck and Ruprecht.

[6] KÄSTNER, Abraham Gotthelf (1759, ^{2}1765, ^{3}1781, ^{4}1792): *Anfangsgründe der angewandten Mathematik. Astronomie, Geographie, Chronologie und Gnomonik. Der mathematischen Anfangsgründe zweyten Theils zweyte Abtheilung*. Göttingen: Vandenhoeck and Ruprecht.

[7] KÄSTNER, Abraham Gotthelf (1760, ^{2}1767, ^{3}1794): *Anfangsgründe der Analysis endlicher Größen. Der mathematischen Anfangsgründe dritten Theils erste Abtheilung*. Göttingen: Vandenhoeck and Ruprecht.

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[8] KÄSTNER, Abraham Gotthelf (1761, [2]1770, [3]1799): *Anfangsgründe der Analysis des Unendlichen. Der mathematischen Anfangsgründe dritten Theils zweyte Abtheilung*. Göttingen: Vandenhoeck and Ruprecht.

[9] KÄSTNER, Abraham Gotthelf (1766, [2]1793): *Anfangsgründe der höhern Mechanik. Der mathematischen Anfangsgründe vierten Theils erste Abtheilung*. Göttingen: Vandenhoeck and Ruprecht.

[10] KÄSTNER, Abraham Gotthelf (1769, [2]1797): *Anfangsgründe der Hydrodynamik. Der mathematischen Anfangsgründe vierten Theils zweyte Abtheilung*. Göttingen: Vandenhoeck and Ruprecht.

KARSTEN, Wenceslaus Johann Gustav (1780): *Anfangsgründe der mathematischen Wissenschaften,* three volumes. Greifswald: Röse.

KARSTEN, Wenceslaus Johann Gustav (1781, [2]1785): *Auszug aus den Anfangsgründen und dem Lehrbegriffe der mathematischen Wissenschaften*, second edition in two volumes. Greifswald: Röse.

KARSTEN, Wenceslaus Johann Gustav (1758-1761): *Beyträge zur Aufnahme der Theoretischen Mathematik*, four volumes. Rostock: Röse.

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KLÜGEL, Georg Simon ([2]1792/93, [3]1798, [7]1821): *Anfangsgründe der Arithmetik, Geometrie und Trigonometrie, nebst ihrer Anwendung auf praktische Rechnungen, das Feldmessen und die Markscheidekunst*. Berlin and Stettin: Nicolai.

KLÜGEL, Georg Simon (1793): *Anfangsgründe der Astronomie, nebst der mathematischen Geographie, Schifffahrtskunde, Chronologie und Gnomonik*. Berlin and Stettin: Nicolai

KLÜGEL, Georg Simon (1784): *Anfangsgründe der praktischen Mechanik, der bürgerlichen Baukunst und der Kriegsbaukunst*. Berlin and Stettin: Nicolai.

MASSENBACH, Christian Karl August Ludwig von (1784): *Anfangsgründe der Differenzial- und Integral-Rechnung zum Gebrauch des Ingenieurs und Artilleristen*. Halle: Gebauer.

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